3.2. Compound Interest

Wednesday, January 24, 2018 1:14 PM

Goals: (1) Understand and apply the Compound Interest Formula

(2) Understand and apply the annual percentage yield (APY) formula.

What is the difference between compound interest and simple interest?

E.g. Put $100 in a bank account

Annual Interest Rate of 6%.

Compound Interest, compounded monthly.

→ Monthly interest rate $= \frac{6\%}{12} = 0.5\% = 0.005$

End of 1st month: $\$100 + \$100 \cdot (0.005)$

$= \$100 (1 + 0.005)$

$= \$100 \cdot (1.005)$

End of 2nd month:

$\$100 \cdot (1.005) + \$100 \cdot (1.005) \cdot (0.005)$

$= \$100 \cdot (1.005)\left[1 + 0.005\right]$

$= \$100 \cdot (1.005) \cdot (1.005) = \$100 \cdot (1.005)^2$

End of 3rd month: $\$100 \cdot (1.005)^3$

⋮

End of 12th month: $\$100 \cdot (1.005)^{12} = 106.17$

Compound Interest Formula:

$$A = P\left(1 + \frac{R}{m}\right)^{mt}$$

A : final amount

P : principal

R : annual interest rate

m : # of compounding periods per year.

t : # of years

A : final amount

P : principal

R : annual interest rate

m : # of compounding periods per year

t : # of years

Let $i = \frac{R}{m}$ = interest rate per compounding period

$n = mt$ = total # of compounding periods.

$$A = P(1+i)^n$$

E.x. Bank pays 6% annual interest compounded semiannually. You want to have \$8000 after 4 years. How much money should you deposit now?

$$A = P\left(1+\frac{R}{m}\right)^{mt}$$

$$\$8000 = P\left(1+\frac{0.06}{2}\right)^{2\cdot 4}$$

$$\$8000 = P(1+0.03)^8 = P\cdot(1.03)^8$$

$$P = \frac{\$8000}{(1.03)^8} = \boxed{\$6315.27}$$

E.g. How long does it take for $2000 to grow to $22000 if it is invested in an account that compounds monthly with an annual interest rate of 7%?

Natural logarithm and the # e.

$$e \approx 2.71828$$