E.g. How long does it take for \$2000 to grow to \$22000 if it is invested in an account that compounds monthly with an annual interest rate of 7%?

Natural logarithm and the # e.

$$e \approx 2.71828$$

$$A = P(1+i)^n$$

$$22000 = 2000\left(1+\frac{0.07}{12}\right)^n$$

$$22000 = 2000(1.00583)^n$$

→ Find n.

$$11 = (1.00583)^n$$

Take LN of both sides:

$$LN(11) = n \cdot LN(1.00583)$$

$$n = \frac{LN(11)}{LN(1.00583)}$$

$$n \approx 412.5$$

$n = m \cdot t \longrightarrow 412.5 = 12t$

$$t = \frac{412.5}{12} = 34 \text{ years.}$$

Annual Percentage Yield (APY)

The annual percentage yield (APY) is the simple interest rate that will produce the same amount as a given compound interest rate in a year.

E.g. compound interest
Compound monthly
Annual interest rate
is 5%

Bank #1

compound interest
compound daily
annual interest rate
is 4.5%

Bank #2

$P = \$1$; $t = 1$ year.

$$A = P(1+i)^n$$

Bank 1: $A = \left(1 + \frac{0.05}{12}\right)^{12} = \$1.05116$

Bank 2: $A = \left(1 + \frac{0.045}{360}\right)^{360} = \$1.04602$

Formula for APY:

$$APY = \left(1 + \frac{R}{m}\right)^m - 1$$

Bank 1: $APY = \left(1 + \frac{0.05}{12}\right)^{12} - 1 = 0.05116$

$\longrightarrow 5.1\%$

APY Bank 2: $\left(1 + \frac{0.045}{360}\right)^{360} - 1 = 0.04602$

$\longrightarrow$ 4.6%

Formula for continuously compound interest.

$$A = P \cdot e^{Rt}$$

where $e \approx 2.71828$

E.g. $P = \$100$. $R = 5\%$. $t = 5$ years.

compounded continuously.

$A = \$100 \cdot e^{(0.05)\cdot 5} = \$128.4$