# 3.3. Future Value of an Annuity, Sinking Funds

Monday, January 29, 2018 1:01 PM

**Goals:** (1) Compute the future value of an annuity

(2) Solve problems that involve sinking funds.

What is an annuity?

An annuity is a sequence of equal periodic payments.

E.g. Retirement account.

Contribution: $500 every month.

company match: $500

**Total:** $1000 every month to the account

This $1000 is invested in a mutual fund compounded monthly at an annual interest rate of 7% for 30 years.

**Q:** How much money in your account after 30 years?

Here's the formula to calculate the final amount:

$$FV = PMT \cdot \left( \frac{(1+i)^n - 1}{i} \right)$$

FV = future value (the final amount in your account)

PMT = periodic payment

$i = \frac{R}{m}$ = interest rate per compounding period

$n = mt$ = total # of compounding periods

---

$$FV = \$1000 \cdot \left( \frac{\left(1 + \frac{0.07}{12}\right)^{360} - 1}{\frac{0.07}{12}} \right)$$

$$= \$1,219,971$$

Amount being contributed = $\$1000 \cdot 12 \cdot 30 = 360000$

Interest you have earned in this period

$$= FV - (\text{amount contributed})$$

$$= \$1219971 - 360000 = 859971$$

Interest earned = $FV - PMT \cdot n$

## Sinking Funds.

**Annuity:** given the amount of periodic payment (PMT)

→ Calculate FV.

**Sinking fund:** Given FV (target amount)

→ Find how much you should contribute periodically to the account.

(find PMT)

Derivation of the formula:

$$FV = PMT\left(\frac{(1+i)^n - 1}{i}\right)$$

**Goal:** Find PMT?

$$PMT = \frac{FV}{\frac{(1+i)^n - 1}{i}}$$

$$PMT = FV \cdot \left( \frac{i}{(1+i)^n - 1} \right)$$

E.g. Set up college fund.

Goal: Want to have \$100,000 at the child's 18th birthday.

Deposit some amount $r$ (quarterly) into an account for 18 years

annual interest rate is 7% compounded quarterly

How much should you contribute every quarter?

$$PMT = 100{,}000 \left( \frac{\frac{0.07}{4}}{\left(1 + \frac{0.07}{4}\right)^{72} - 1} \right)$$

$$\approx \$703.6$$

Ex. Bob contributed $2500 into a retirement account yearly for 26 years.

The account has an annual interest rate of 6.4%. It is compounded yearly.

Then he stops contribution.

Leave the money in the account for the next 16 years. During this time, account was restructured interest rate is 7%, compounded monthly.

Q: How much money is in the account when Bob withdraw it for retirement?

How much interest has he earned.

First 26 years:

$$FV = PMT\left(\frac{(1+i)^n - 1}{i}\right)$$

$$= 2500 \cdot \left(\frac{(1+0.064)^{26} - 1}{0.064}\right)$$

$$\approx \$156\,931.31$$

Next 16 years:

$$A = P(1+i)^n$$

$192$ / $12 \cdot 16$

$$A = \$156931.31\left(1 + \frac{0.07}{12}\right)$$

$$\approx \$479\,409.04$$

Total interest earned $= 479409.04 - 2500 \cdot 26$

$$= 414\,409.04.$$