3.4 Present Value of an Annuity, Amortization

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Goals: (1) Calculate the present value of an annuity

(2) Calculate payment on a loan.

(3) Construct an amortization table.

Recall: Last time, future value of an annuity.

$$FV = PMT\left(\frac{(1+i)^n - 1}{i}\right)$$

Future value: We make periodic payments now, how much will the account be worth in the future?

→ Now, if we want to withdraw periodic amount in the future, what amount should be deposited now?

E.g. A lending service offers 42-month loan at 6.6% annual interest rate compounded monthly. You can afford payment of $225/month.

Q: How much money can you borrow now?

Present Value Formula:

$$PV = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$$

$\rightarrow$ present value

PMT: periodic payment.

$i = \frac{R}{m}$ = rate per period

$n = mt$ = # of periods

$$PV = \$225 \cdot \left(\frac{1-\left(1+\frac{0.066}{12}\right)^{-42}}{\frac{0.066}{12}}\right) \approx \boxed{8417.37}$$

Amortization Problem (loan payment)

A amortization of a debt is the process of paying it off in equal payments.

E.g. Bank loans you $250 000 at 3% annual interest rate to pay for a house. You agree to make monthly payments for the next 15 years. How much should the monthly payment be?

$$PV = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$$

Want: PMT ?

$$PMT = \frac{PV}{\frac{1-(1+i)^{-n}}{i}}$$

$$PMT = PV \cdot \left(\frac{i}{1 - (1+i)^{-n}} \right)$$

$$PMT = 250000 \cdot \left(\frac{\frac{0.03}{12}}{1 - \left(1 + \frac{0.03}{12}\right)^{-180}} \right)$$

$$\approx \$1726.25$$

How much interest did the bank earn?

$$\text{Interest} = (1726.25) \cdot (180) - 250000$$

$$= \$60761.74$$

Ex. 30 year plan. $R = 3.4\%$.

Monthly payment? \$1108.7

Total interest? 149132

$$PMT = 250000\left(\frac{\frac{0.034}{12}}{1-\left(1+\frac{0.034}{12}\right)^{-360}}\right)$$

$$\approx 1108.7$$

How much of each monthly payment goes toward interest? How much goes towards reducing the unpaid balance.

30 year plan. Monthly payment. $1108.7

$250000 loan.

End of 1st month:

Interest Due: $(250{,}000)\cdot\left(\frac{0.034}{12}\right) \approx 708.33$

Balance reduction amount: $1108.7 - 708.33$

≈ 400.37

Unpaid balance after 1st month:

$250000 - 400.37 = \$249599.63$

End of 2nd month:

Interest due $= (249599.63)\cdot\left(\frac{0.034}{12}\right)$

$\approx \$707.2$

Balance reduction amount:

$1108.7 − $707.2 = $401.5