4.3. Gauss-Jordan Elimination

Tuesday, October 3, 2017 8:36 AM

Goals: (1) Reduced Row-Echelon form of a matrix

(2) Gauss-Jordan Elimination.

E.g.

$$\begin{aligned} x + 2y - z &= 5 \\ 2x - 3y + z &= 3 \\ 3x - y - 2z &= 7 \end{aligned}$$

Augmented matrix?

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 2 & -3 & 1 & 3 \\ 3 & -1 & -2 & 7 \end{array}\right)$$

Some possible "nice" form of a 3-by-3 matrix that will tell us what the solution(s) to system look like.

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & \boxed{1} & 0 & 7 \\ 0 & 0 & 1 & 8 \end{array}\right)$$

$x = 6$
$y = 7$
$z = 8$

$$\left(\begin{array}{ccc|c} \boxed{1} & 2 & 3 & 4 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

No Solution

$$\left(\begin{array}{ccc|c} \boxed{1} & 2 & 0 & 4 \\ 0 & 0 & \boxed{1} & 5 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Infinitely many solutions

Reduced Row Echelon form.

A matrix is in Reduced Row echelon form if:

1. Each row consisting entirely of zeros must be below any row having a nonzero entry
2. The leftmost nonzero element in each row must be 1.

(3) All other elements in the column that contains the leftmost 1 of a given row must be zero.

(4) The leftmost 1 in any row is to the right of the leftmost 1 in the row above it.

E.g.

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -18 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$R_1 \leftrightarrow -2R_2 + R_1$

$R_1 \leftrightarrow -3R_3 + R_1$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 1 & 0 & 4 \end{array}\right)$$

Not in reduced row echelon form $\longrightarrow R_2 \leftrightarrow R_3$.

E.g. Use Gauss-Jordan elimination to turn the given matrix into reduced row-echelon form.

Tuesday, October 3, 2017 9:06 AM

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 2 & -1 & 1 & 5 \\ -1 & 2 & 2 & 1 \end{array}\right) \xrightarrow[R_2 \leftrightarrow -2R_1 + R_2]{}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ -1 & 2 & 2 & 1 \end{array}\right) \xrightarrow[R_3 \leftrightarrow R_1 + R_3]{} \left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ 0 & 3 & 1 & -1 \end{array}\right)$$

$$\xrightarrow[R_2 \leftrightarrow -\frac{1}{3}R_2]{} \left(\begin{array}{ccc|c} 1 & \boxed{1} & -1 & -2 \\ 0 & \boxed{1} & -1 & -3 \\ 0 & \boxed{3} & 1 & -1 \end{array}\right) \xrightarrow[R_1 \leftrightarrow -R_2 + R_1]{} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -3 \\ 0 & 3 & 1 & -1 \end{array}\right)$$

$$\xrightarrow[R_3 \leftrightarrow -3R_2 + R_3]{} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 4 & 8 \end{array}\right) \xrightarrow[R_3 \leftrightarrow \frac{1}{4}R_3]{} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$$\xrightarrow[R_2 \leftrightarrow R_3 + R_2]{} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$x = 1$

$y = -1$

$z = 2$