5.2. System of linear Inequalities in 2 Variables

E.g.
$$\begin{cases} x + y > 6 \longrightarrow \boxed{y > -x + 6} \\ 2x - y > 0 \longrightarrow -y > -2x \\ \boxed{y < 2x} \end{cases}$$

Solve this system.

$(2,4)$?

corner point

Solution.

Solution Region
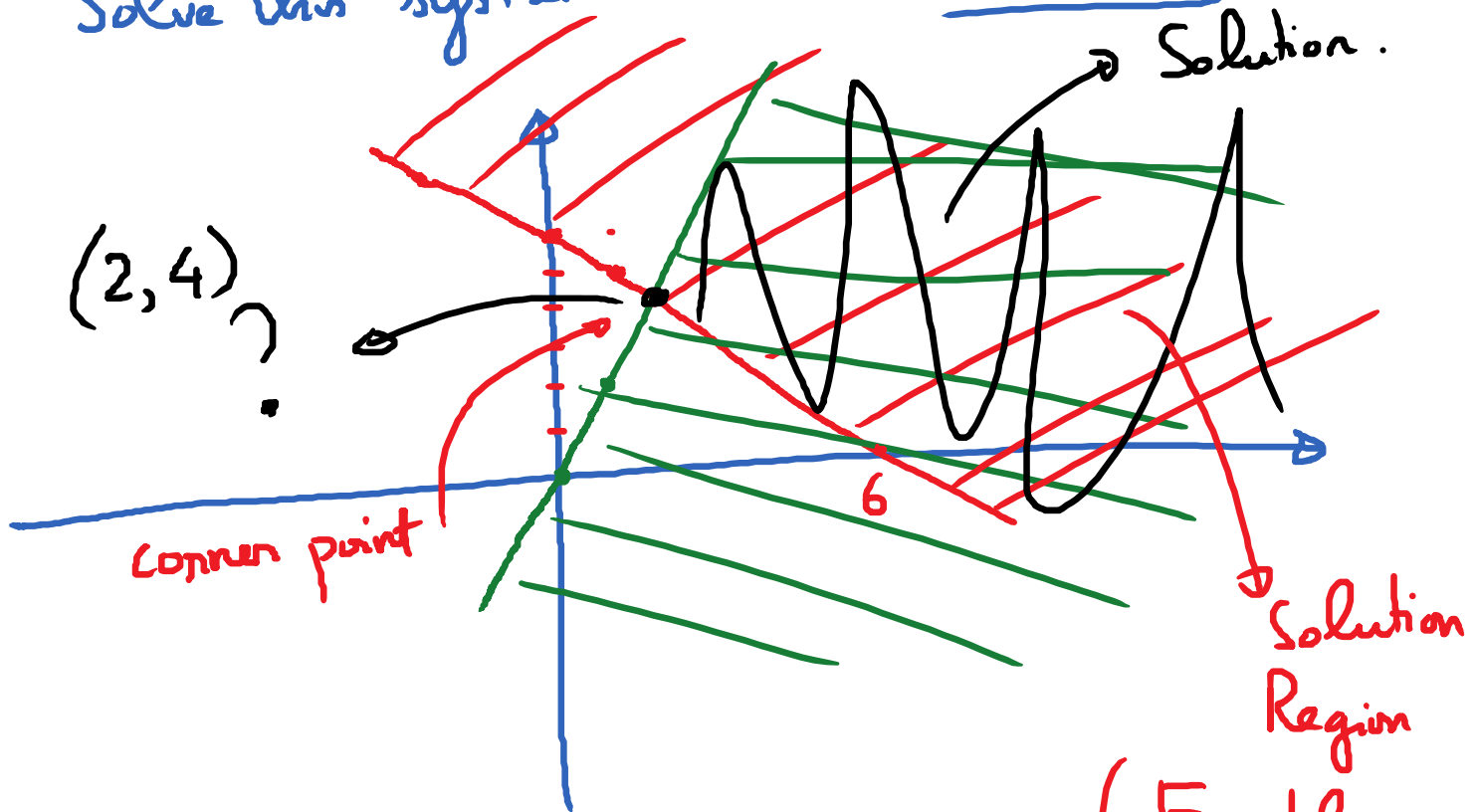
(Feasible region)

The solution region for a system of linear inequalities is called the feasible region of the system. In this example, the feasible region is unbounded.

- A corner point is the intersection of 2 boundary lines of the feasible region.

Application:

A manufacturer produces 2 kinds of product.

A: 8 hours to design, 4 hours to finish.

B: 8 hours to design, 12 hours to finish.

Total # of hours to design products is at most

160 hours.

Total # of hours to finish products is at most

180 hours.

The # of products of type A is no more than 15.

Let the variable x to represent the # of units of A

y _____ B.

Q: ① Write down a system of inequalities that describe all the constraints that x and y must satisfy.

② Solve the inequalities by finding the feasible region and the corner points.

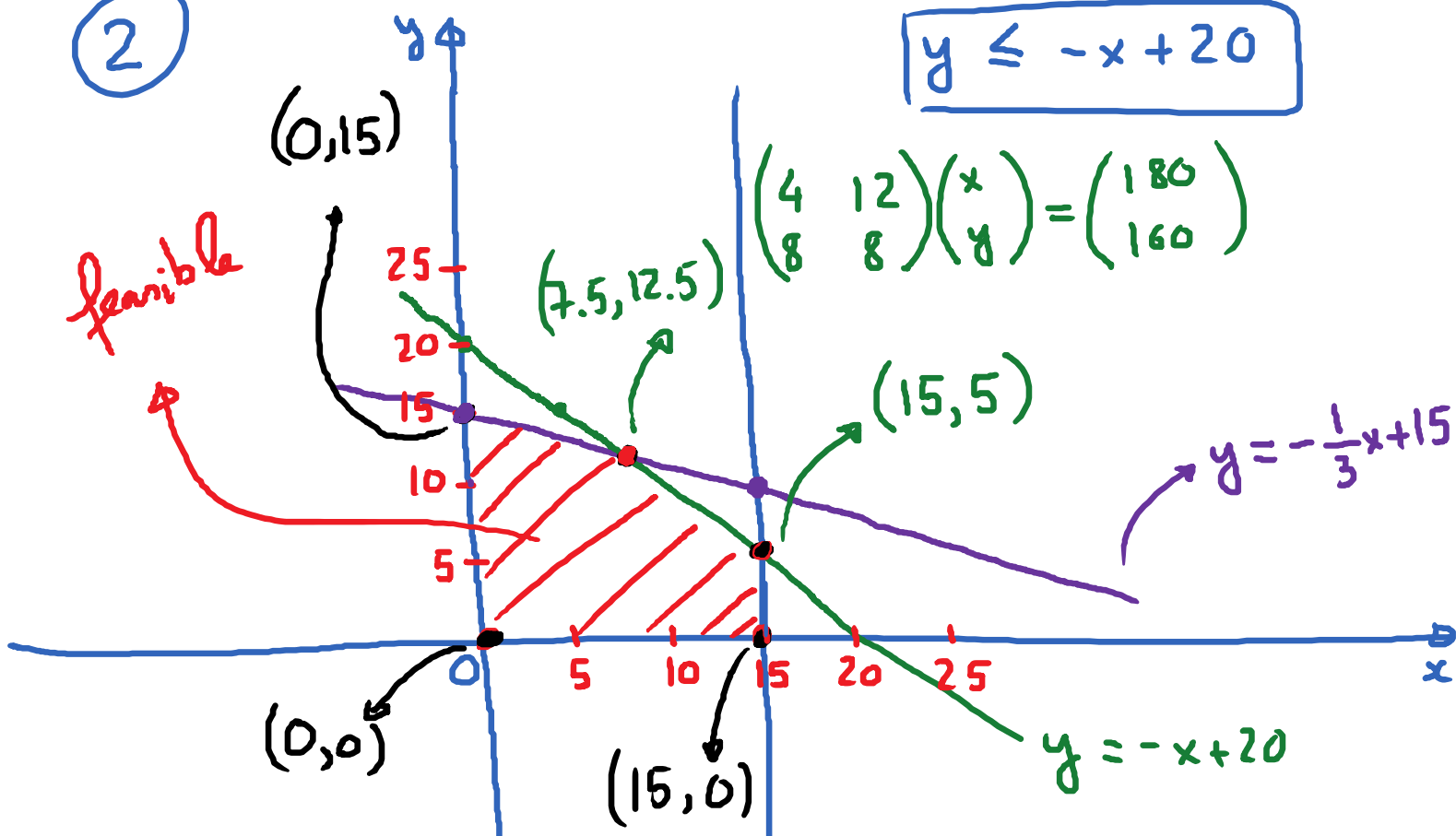---

① $\boxed{x \leq 15}$

$x \geq 0$

$y \geq 0$

$\boxed{4x + 12y \leq 180}$ $\quad\longrightarrow\quad$ $12y \leq 180 - 4x$

$\quad y \leq 15 - \dfrac{4x}{12}$

$8x + 8y \leq 160$ $\quad\longrightarrow\quad$ $8y \leq 160 - 8x$

$\boxed{y \leq -\dfrac{1}{3}x + 15}$

$\boxed{y \leq -x + 20}$

②

(0,15)

feasible

$\begin{pmatrix} 4 & 12 \\ 8 & 8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 180 \\ 160 \end{pmatrix}$

(7.5, 12.5)

(15, 5)

$y = -\dfrac{1}{3}x + 15$

(0,0)

(15,0)

$y = -x + 20$

(−3,−) $(15, 0)$ $y = x + 20$

Ex. A psychologist uses 2 types of of boxes when
    studying mice and rats.
    Each mouse spends: 10 mins a day in box A
                        20 mins a day in box B
    Each rat spends : 20 mins a day in box A
                       10 mins a day in box B.

Total maximum time available per day is 820 mins
for A. and 560 mins for B.
Let x be the # of mice and y be the # of rats.
Q:* Find a system of inequalities for x and y.
    * Solve the system to find the feasible region
        * corner points.

$$10x + 20y \leq 820 \longrightarrow y \leq -\frac{1}{2}x + 41$$
$$20x + 10y \leq 560 \longrightarrow y \leq -2x + 56$$
$$x \geq 0$$
$$y \geq 0$$