7.2. Sets

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Goals: (1) Understand Set Notations

(2) Perform Operations on Sets

(3) Solve some applications.

Definition of a set:

A set is a collection of objects

E.g. (finite sets)

- $A = \{1, 2, 3\}$ ← listing method
- $B = \{a, b, c, d, e, f\}$
- $C = \{\text{Jan}, \text{Feb}, \text{Mar}, \text{May}, \text{Jun}, \text{July}\}$

- $\mathbb{R}$ = set of all real numbers
- $\mathbb{Z}$ = set of all integers

} examples of infinite sets.

Notation: $a \in A$

This notation means that a is an element of the set A.

$A = \{1, 2, 3\}$

$1 \in A$, $2 \in A$, $3 \in A$

read as 1 is in A or 1 belongs to A; etc

$B = \{$ all states in U.S. $\}$

$TX \in B$, $NY \in B$, etc.

Russia $\notin B$

read as Russia is not in B

Notation: $a \notin A$.

This means a is NOT an element of A or a is NOT in A.

Empty Set or Null Set (set with no elements)

Notation: $\emptyset$

E.g. Find the set of all real solutions to the equation $x^2 + 1 = 0$

This equation has no real solutions.

So the set of real solutions is empty. It can be denoted by $\emptyset$

Set Builder Notations

$A = \{a, b, c, d, \ldots, z\}$ ⟶ listing method.

Instead of using the listing method, we can use set builder notation to describe sets.

$$A = \{ x \mid x \text{ is a letter in the English alphabet} \}$$

read as "such that"

verbal description of the set

$B = \{ x \mid x \text{ is a month in a year} \}$

$C = \{ x \mid x \text{ is a rational number} \}$

Subsets:

Definition: A and B are sets.

We say that A is a subset of B if every element of A is also an element of B.

$A = \{ x \mid x \text{ is a real \#} \}$

$B = \{ x \mid x \text{ is a negative \#} \}$

Is A a subset of B? No

$A = \{ x \mid x \text{ is a negative \#} \}$
$B = \{ x \mid x \text{ is a real \#} \}$
} A is a subset of B

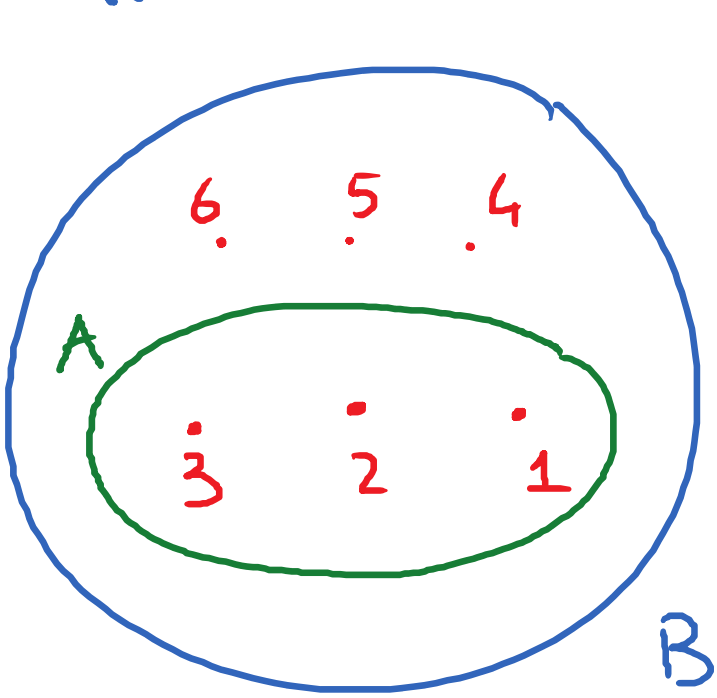
E.g. $A = \{3, 2, 1\}$

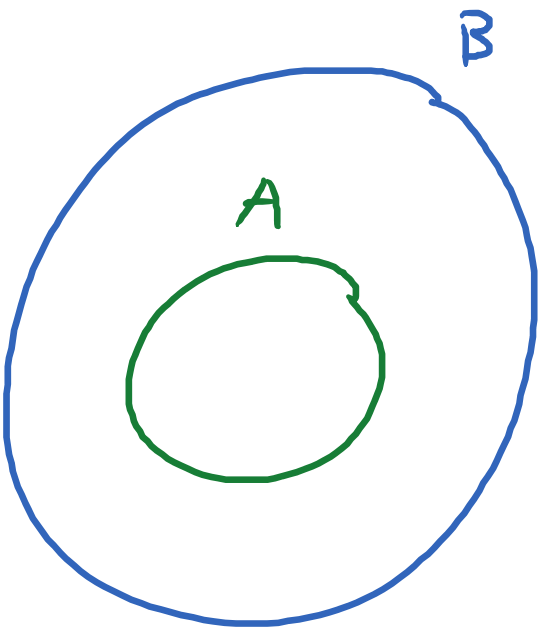
$B = \{6, 5, 4, 3, 2, 1\}$

→ A is a subset of B.

Notation for subset: $A \subset B$

To visualize sets and the relations among sets, we can use Venn Diagram.





$A \subset B$.

Note: the empty set is a subset of any set

Ex. $A = \{1, 2, 3\}$

List all the subsets of A.

$\{1,2,3\}$; $\{1,2\}$; $\{1\}$; $\{2\}$; $\{3\}$
$\{2,3\}$; $\{1,3\}$; $\emptyset$ → A has 8 subsets.

($\{1,2\} \subset A$, etc. . .)

Ex. $B = \{1, 2, 3, 4\}$. Find all the subsets of B.

$\{1\}$; $\{2\}$; $\{3\}$; $\{4\}$
$\{1,2\}$; $\{1,3\}$; $\{1,4\}$; $\{2,3\}$; $\{3,4\}$;
$\{2,4\}$; $\emptyset$; $\{1,2,3\}$; $\{1,2,4\}$; $\{2,3,4\}$
$\{1,3,4\}$; $\{1,2,3,4\}$

→ 16 subsets

Ex. $C = \{1, 2, 3, 4, 5\}$ → How many subsets does C have?

32 subsets