7.4. Permutations and Combinations

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Goals:
1. Compute Factorials
2. Apply Permutations
3. Apply Combinations

1 Factorials:

E.g. Notation $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$4!$ read as 4 factorial

So, $4! = 24$

$$5! = 5 \cdot \underbrace{4 \cdot 3 \cdot 2 \cdot 1}_{24} = 5 \cdot 24 = 120$$

$$5! = 5 \cdot (4!)$$

In general, $n!$ (read as n factorial) is equal to the product of the first n whole numbers.

$$\boxed{n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdots 1}$$

Note: $n! = n \cdot [(n-1)!]$

Note: $0! = 1$

Do some calculations with factorials.

E.g. $16! \longrightarrow$ calculator, large number.

Calculate: $\dfrac{16!}{15!} = \dfrac{16 \cdot \cancel{15} \cdot \cancel{14} \cdots \cancel{1}}{\cancel{15} \cdot \cancel{14} \cdots \cancel{1}} = \boxed{16}$

$$\frac{16!}{14!} = 16 \cdot 15 = 240$$

Ex. **Calculate:** $\dfrac{100!}{(98!) \cdot (2!)} = 4950$

$$= \frac{100 \cdot 99}{2} = 4950$$

② Permutations

E.g. Group of 5 people: A, B, C, D, E.
From these 5, select a committee of 3 consisting of 1 President, 1 VP, 1 Treasurer.
How many different committees can you select?

$$\underset{P}{\boxed{5}} \cdot \underset{VP}{\boxed{4}} \cdot \underset{T}{\boxed{3}} = 60$$

→ Situation: you want to select a subset of 3 elements from a set of 5 elements.
P VP T

Order matters. (the choice A, B, C is different from the choice C, B, A)
P VP T

→ A permutation problem.

The solution to this permutation problem is denoted by $\underline{P}(5,3)$

$$\underline{P}(5,3) = 60$$

In general, if you have n objects and you want to select r objects from these, and the order matters, then the # of ways you can do it is

$$\underline{P}(n,r).$$

Formula for $\underline{P}(n,r) = \dfrac{n!}{(n-r)!}$

$\underline{\text{E.g.}}$ $\underline{P}(5,3) = \dfrac{5!}{2!} = 3 \cdot 4 \cdot 5 = 60$

E.g. 15 different local music bands

Invite 4 to come to campus to perform.

1 performs in the student center

1 performs in the theater

1 performs in the courtyard.

1 performs in the cafeteria.

How many different invitations can be sent out?

→ choose 4 objects from 15 objects where order matters.

→ Answer: $P(15,4) = 32760.$

What if the order doesn't matter?

3 Combinations

E.g. Group of 5 people: A, B, C, D, E.

Invite 3 people from this group to dinner.

→ Choose 3 objects from 5 objects where the order does not matter.

→ Combination: $C(5,3) = 10$

In general, if you want to choose r objects from n objects, where the order does not matter, then the # of ways to do this is given by Combination $C(n,r)$.

Formula for $C(n,r)$ is :

$$C(n,r) = \frac{n!}{(n-r)!\,r!}$$

$$C(5,3) = \frac{5!}{2!\,3!} = \frac{4 \cdot 5}{2} = 10.$$

Ex. 1 State lottery:

Select 6 numbers from 49 numbers.

To win the lottery, you must have the correct set of 6 numbers.

Q : How many different lottery tickets are there?

$\longrightarrow$ order doesn't matter

$\longrightarrow$ $C(49,6) = 13\,983\,816$