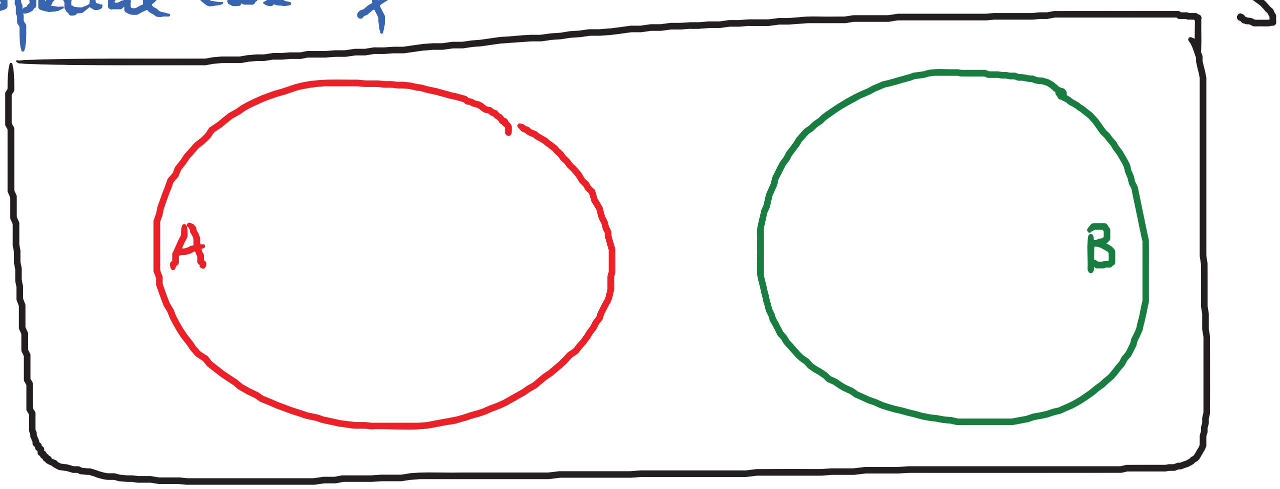
* Special Case of the Addition Rule:



$$A \cap B = \emptyset$$

(We say that A and B are mutually exclusive if the intersection is empty)

$$\underline{P}(A \cap B) = 0$$

$$\boxed{\underline{P}(A \cup B) = \underline{P}(A) + \underline{P}(B)} - \cancelto{0}{\underline{P}(A \cap B)}$$

E.g. Toss 2 fair coins once.

A : event that you get at least 1 H

$$A = \{HH, HT, TH\}$$

B : event that you get exactly 2T.

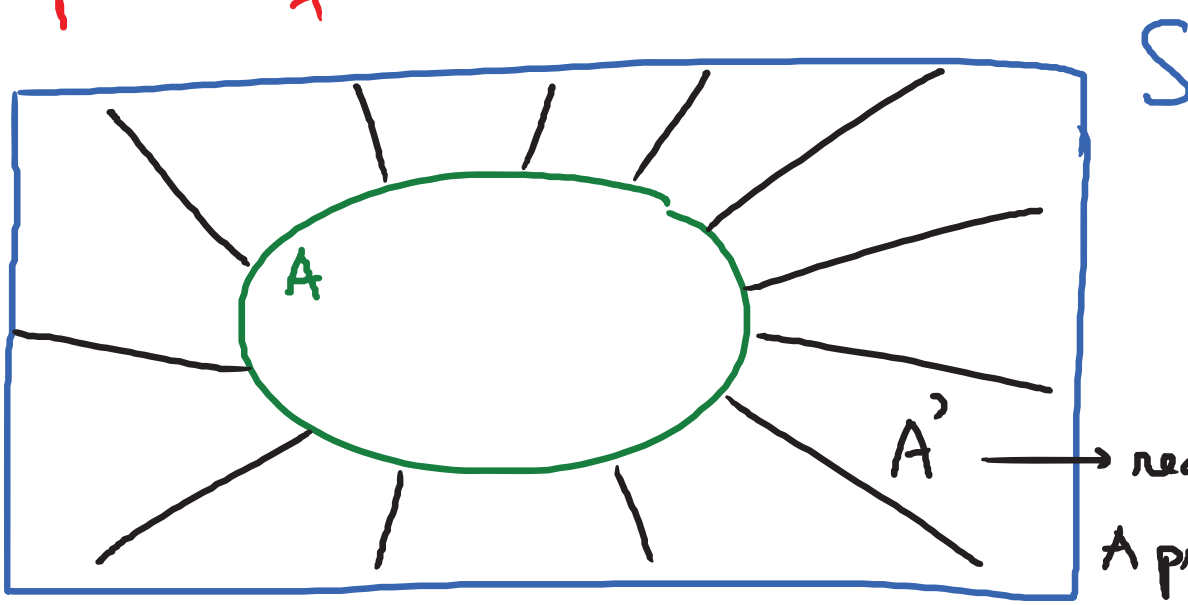
$$B = \{TT\}$$

So, $A \cap B = \emptyset$

$$\underbrace{P(A \cup B)}_{1} = P(A) + P(B)$$

$$1 = \frac{3}{4} + \frac{1}{4}$$

The complement of an event.



$\rightarrow$ read as A prime

$$P(A) + P(A') = 1$$

$$\longrightarrow \boxed{P(A') = 1 - P(A)}$$

E.g. Experiment: Toss 2 dice.

Q: Find the probability that the # of points on each dice are not the same?

Sol: Let A be the event that the # of points on the dice are the same.

We are interested in A' and P(A')

$$A = \{(1,1), \ldots, (6,6)\} \rightarrow P(A) = \frac{6}{36}.$$

$$\text{So, } P(A') = 1 - \frac{6}{36} = \frac{30}{36} = \boxed{\frac{5}{6}}.$$

E.g. Toss 2 dice.

Q: Find the probability that the sum of 2 dice > 4.

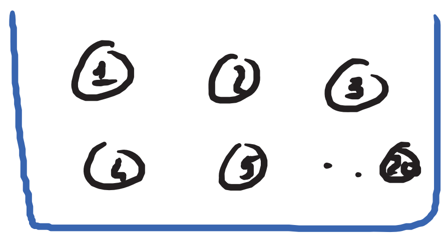
A : event that sum ≤ 4

$$A = \{(1,1),(2,2),(2,1),(1,2),(3,1),(1,3)\}$$

$$P(A) = \frac{6}{36};$$

So, $P(A') = 1 - \frac{6}{36} = \frac{30}{36} = \boxed{\frac{5}{6}}$.

Ex. A bag contains 20 balls numbered from 1 to 20



Select 2 balls at the same time randomly from bag.

Q: Find the probability that the two numbers selected do not differ by 12.

Number of elements in sample space $= \frac{20 \cdot 19}{2}$.

$= 190$.

A : 2 balls differ by 12 :

1 − 13, 2 − 14, . . . , 8 − 20

$P(A) = \frac{8}{190}$.

$P(A') = \frac{182}{190} = \boxed{\frac{91}{95}}$.

E.g. Take a fair 6-sided dice. Roll it 10 times. Find the probability that the # 1 is rolled at least once.

$$\text{Prob} = 1 - \left(\frac{5}{6}\right)^{10}$$

Odds against and odds in favor of an event.

E: event.

The odds in favor of E = (ratio success : failure)

$$= \frac{P(E)}{P(E')}$$

The odds against E = $\frac{P(E')}{P(E)}$ (ratio of failure : success)

E.g. Toss 2 fair coins.

E = get a least 1H = $\{HT, TH, HH\}$

$E' = \{TT\}$

Calculate the odd in favor of E = $3 : 1$

the odd against E = $1 : 3$.