Ex. Rare Disease: found in 0.5% of the population.

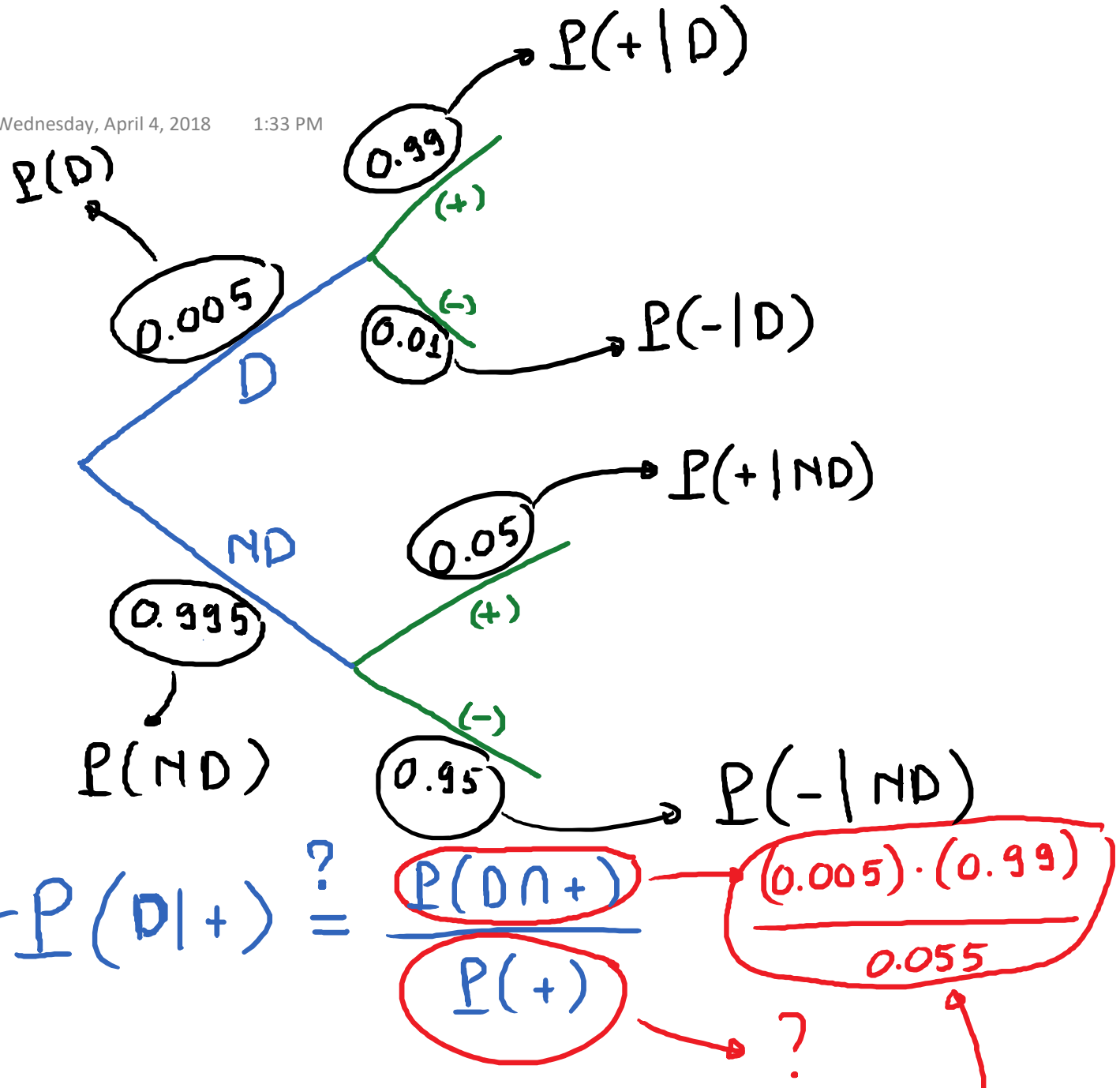
Blood Test for this disease:

* The test is 99% accurate if the disease is present.
* The test has a 5% false positive rate.

(if disease is not present, there is still 5% positive result)

Q: If one gets a positive result, find the probability that one actually has the disease?

Wednesday, April 4, 2018 1:33 PM



$$P(D|+) \overset{?}{=} \frac{P(D \cap +)}{P(+)} = \frac{(0.005)\cdot(0.99)}{0.055}$$

$P(+)$?

$$P(+) = (0.005)\cdot(0.99) + (0.995)\cdot(0.05) = 0.055$$

So, $P(D|+) = 0.091 \rightarrow$ **9.1%**

Find $P(ND|-) \overset{?}{=} \frac{P(ND \cap -)}{P(-)} = \frac{(0.995)\cdot(0.95)}{(0.005)(0.01) + (0.995)\cdot(0.95)}$

Independent Events:

A and B are events. We say that A and B are independent if:

1. $P(B|A) = P(B)$
2. $P(A|B) = P(A)$

Note: these 2 equations give rise to a single equation that can be used to test whether 2 events are independent.

$$P(B|A) = P(B)$$

$$\rightarrow \frac{P(A \cap B)}{P(A)} = P(B)$$

$$\rightarrow \text{Multiply both sides by } P(A):$$

$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

* $P(A|B) = P(A)$

$$\frac{P(A \cap B)}{P(B)} = P(A)$$

$$\rightarrow \boxed{P(A \cap B) = P(A) \cdot P(B)}$$

So, 2 events A and B are independent if

$$\boxed{P(A) \cdot P(B) = P(A \cap B)}$$

E.g. Pick a card at random from a standard 52 card deck

A = event that we get a spade or a club.

B = event that we get a spade or a heart

Are A and B independent? $P(A) = \frac{1}{2}$

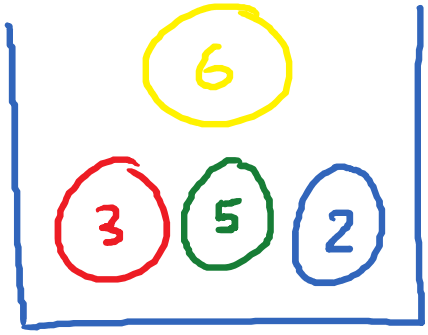
$P(B) = \frac{1}{2}$.

$A \cap B$ = event that we get a spade

$$P(A \cap B) = \frac{13}{52} = \frac{1}{4}.$$

$$\underbrace{P(A)}_{\frac{1}{2}} \cdot \underbrace{P(B)}_{\frac{1}{2}} = \underbrace{P(A \cap B)}_{\frac{1}{4}} \quad \checkmark$$

E.g. Experiment:



jar

contains 3 red, 5 green, 2 blue, 6 yellow marbles

* Pick a marble at random from the jar
* Replace it in the jar
* Pick another marble at random from the jar.

→ Find the probability that we get a green and a yellow marble.

G : event that we get a green in first draw

$$P(G) = \frac{5}{16}$$

Y : event that we get a yellow in second draw.

$$P(Y) = \frac{6}{16}.$$

G and Y are independent.

So, $P(G \cap Y) = P(G) \cdot P(Y)$

prob that we get a green and a yellow $= \frac{5}{16} \cdot \frac{6}{16} = \frac{30}{256} = \frac{15}{128}$

Eg. Different Experiment: (Same jar)

(1) Pick a marble at random. (No Replacement)

(2) Pick another marble at random.

Find Prob that we get a green and a yellow.

$$P(G \cap Y)$$

Here $P(G \cap Y) = P(G) \cdot P(Y|G)$

$= \frac{5}{16} \cdot \frac{6}{15} = \frac{30}{240} = \boxed{\frac{1}{8}}$.

Dependent Events:

Events A and B are dependent if they are not independent.

In other words, $P(A \cap B) \neq P(A) \cdot P(B)$

$\left(P(A \cap B) = P(A) \cdot P(B|A) \right)$