(1) $8x_1 + 9x_2 = 117$
$4x_1 + 6x_2 = 66$ $\longrightarrow \begin{pmatrix} 8 & 9 \\ 4 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 117 \\ 66 \end{pmatrix}$

(2) # of machines model A : x
B : y
C : z

$3.2x + 5.4y + 2.2z = 303 \rightarrow$ electronic work

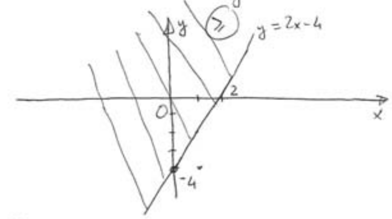
$2.8x + 2.4y + 5.8z = 393 \rightarrow$ assembly

$4.4x + 3.4y + 4.8z = 416 \rightarrow$ quality assurance

We can solve this system using the TI-calculator.

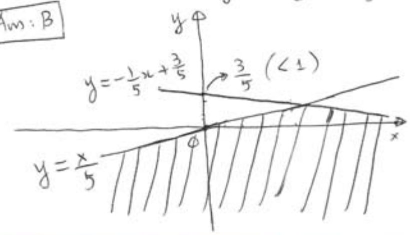
Solution: $x = 30$
$y = 20$
$z = 45$

(3) $4x - 2y \leq 8 \longleftrightarrow -2y \leq -4x + 8$
$\longleftrightarrow y \geq 2x - 4$



(4) $x \geq 5y \longrightarrow y \leq \frac{x}{5}$
$x + 5y \leq 3 \longrightarrow y \leq -\frac{1}{5}x + \frac{3}{5}$

$\boxed{\text{Ans: B}}$



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(5) $x_1 + x_2 \leq 87$
$3x_1 + x_2 \leq 189$

Maximize $z = 2x_1 + x_2$

Slack variables:

$x_1 + x_2 + s_1 = 87$
$3x_1 + x_2 + s_2 = 189$
$-2x_1 - x_2 + z = 0$

$$\begin{array}{c} \\ s_1 \\ s_2 \\ z \end{array}\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & s_1 & s_2 & z & \end{array} \\ \left(\begin{array}{ccccc|c} 1 & 1 & 1 & 0 & 0 & 87 \\ 3 & 1 & 0 & 1 & 0 & 189 \\ -2 & -1 & 0 & 0 & 1 & 0 \end{array}\right) \end{array}$$

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(6) enter

$$\begin{array}{c} \text{exit} \\ s_1 \\ s_2 \\ z \end{array}\begin{array}{c} \begin{array}{ccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & z & \end{array} \\ \left(\begin{array}{cccccc|c} 2 & 1 & 4 & 1 & 0 & 0 & 48 \\ 2 & 4 & 1 & 0 & 1 & 0 & 32 \\ -1 & -3 & -2 & 0 & 0 & 1 & 0 \end{array}\right) \end{array}$$

To pivot once around 2, we need to turn 2 to 1 first, then turn everything else in that column into zeros.

The operations needed are

(1) $R_1 \longleftrightarrow \frac{1}{2}R_1$

(2) $R_2 \longleftrightarrow -2R_1 + R_2$

(3) $R_3 \longleftrightarrow R_1 + R_3$

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(6) (Cont) After implementing these operations using the calculator, we obtained the matrix (Note that the labels are updated as well)

$$\begin{array}{c} \\ x_1 \\ s_2 \\ z \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & s_1 & s_2 & z \end{array} \\ \left(\begin{array}{cccccc|c} 1 & \frac{1}{2} & 2 & \frac{1}{2} & 0 & 0 & 24 \\ 0 & 3 & -3 & -1 & 1 & 0 & -16 \\ 0 & -5/2 & 0 & 1/2 & 0 & 1 & 24 \end{array}\right) \end{array}$$

From the last column and the label, we get

$$x_1 = 24, \quad s_2 = -16, \quad z = 24.$$

Since x_2, x_3, s_1 do not appear on the left column

$$x_2 = x_3 = s_1 = 0$$

(7) Initial Matrix is:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 22 & 15 & 1 \end{pmatrix}$$

Take transpose:

$$A^T = \begin{pmatrix} 1 & 1 & 22 \\ 2 & 1 & 15 \\ 3 & 2 & 1 \end{pmatrix}$$

$\rightarrow$ dual problem:

Maximize $P = 3y_1 + 2y_2$

Subject to: $y_1 + y_2 \leq 22$

$2y_1 + y_2 \leq 15$

$y_1, y_2 \geq 0$

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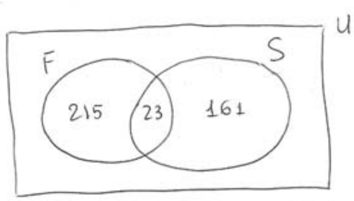
(8)



From the Venn Diagram, # of students who take Finite Math but not Statistics is:

215.

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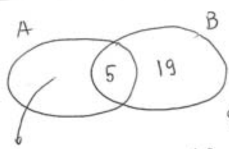
(9) $n(A \cap C) = 2 + 8 = 10$

---

(10)



$? = 42 - 5 - 19 = 18.$

So, $n(A) = 18 + 5$
$= 23$

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(11)

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 & 4 \\ 1 & -2 \end{pmatrix}^{-1} \begin{pmatrix} 25 \\ -11 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

So, $x = -3$, $y = 4$

---

(12) Objective function:

Minimize cost $C = 8500x_1 + 9500x_2$

Constraints:

$3500x_1 + 2500x_2 \geq 8000$ (plain sauce)

$6500x_1 + 2000x_2 \geq 9000$ (sauce with mushrooms)

$3000x_1 + 1500x_2 \geq 6000$ (hot spicy sauce)

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(13) $A \cup B = \{j, n, c, i, e, r\}$

---

(14) $\square\ \square\ \square\ \square\ \square$

10 choices $(0, \dots, 9)$ 10 10 10 10

\# of different codes available: 10^5.

---

(15) Objective function

Maximize Profit $= P = 20x_1 + 30x_2 + 40x_3$

Constraints: $x_1 \leq 100$

$5x_1 + 10x_2 + 15x_3 \leq 2000$

$x_1, x_2 \geqslant 0$

$\longrightarrow$ Introduce slack variables s_1, s_2

(15) Cont

$x_1 \quad + s_1 = 100$

$5x_1 + 10x_2 + 15x_3 + s_2 = 2000$

$-20x_1 - 30x_2 - 40x_3 + P = 0$

Simplex tableau: pivot element

$$\begin{array}{c} \\ s_1 \\ \text{exit } s_2 \\ P \end{array} \begin{array}{c} \begin{array}{ccccccc} x_1 & x_2 & \overset{\text{enter}}{x_3} & s_1 & s_2 & P & \end{array} \\ \left(\begin{array}{cccccc|c} 1 & 0 & 0 & 1 & 0 & 0 & 100 \\ 5 & 10 & 15 & 0 & 1 & 0 & 2000 \\ -20 & -30 & -40 & 0 & 0 & 1 & 0 \end{array}\right) \end{array}$$

1^{st} pivoting $R_2 \longleftrightarrow \frac{1}{15} R_2$

$R_3 \longleftrightarrow R_3 + 40R_2$.

We get pivot element

$$\begin{array}{c} \\ \text{exit } s_1 \\ x_3 \\ P \end{array} \begin{array}{c} \begin{array}{ccccccc} \overset{\text{enter}}{x_1} & x_2 & x_3 & s_1 & s_2 & P & \end{array} \\ \left(\begin{array}{cccccc|c} 1 & 0 & 0 & 1 & 0 & 0 & 100 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{1}{15} & 0 & \frac{400}{3} \\ -\frac{20}{3} & -\frac{10}{3} & 0 & 0 & \frac{8}{3} & 1 & \frac{16000}{3} \end{array}\right) \end{array}$$

2^{nd} pivoting: $R_2 \longleftrightarrow -\frac{1}{3}R_1 + R_2$

$R_3 \longleftrightarrow \frac{20}{3}R_1 + R_3$

(15) Cont.

$$\begin{array}{c} \\ x_1 \\ \text{exit } x_3 \\ P \end{array} \begin{array}{c} \begin{array}{ccccccc} x_1 & \overset{\text{enter}}{x_2} & x_3 & s_1 & s_2 & P & \end{array} \\ \left(\begin{array}{cccccc|c} 1 & 0 & 0 & 1 & 0 & 0 & 100 \\ 0 & \frac{2}{3} & 1 & -\frac{1}{3} & \frac{1}{15} & 0 & 100 \\ 0 & -\frac{10}{3} & 0 & \frac{20}{3} & \frac{8}{3} & 1 & 6000 \end{array}\right) \end{array}$$

3^{rd} pivoting pivot

$R_2 \longleftrightarrow \frac{3}{2}R_2$; $R_3 \longleftrightarrow \frac{10}{3}R_2 + R_3$

We get

$$\begin{array}{c} \\ x_1 \\ x_2 \\ P \end{array} \begin{array}{c} \begin{array}{ccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & P & \end{array} \\ \left(\begin{array}{cccccc|c} 1 & 0 & 0 & 1 & 0 & 0 & 100 \\ 0 & 1 & \frac{3}{2} & -\frac{1}{2} & \frac{1}{10} & 0 & 150 \\ 0 & 0 & 5 & 5 & 3 & 1 & 6500 \end{array}\right) \end{array}$$

Solution: Max $P = 6500$

when $x_1 = 100$, $x_2 = 150$, $x_3 = 0$

(16)

Initial Matrix

$$A = \begin{pmatrix} 3 & 2 & 34 \\ 2 & 5 & 43 \\ 6 & 3 & 1 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 3 & 2 & 6 \\ 2 & 5 & 3 \\ 34 & 43 & 1 \end{pmatrix}$$

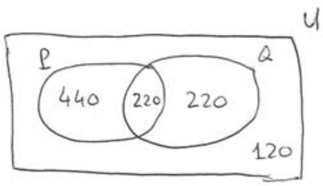
Dual problem

Maximize $34y_1 + 43y_2 = P$

$3y_1 + 2y_2 \leq 6$

$2y_1 + 5y_2 \leq 3$

(17)



From Venn Diagram, # of people who use P and not Q: 440.