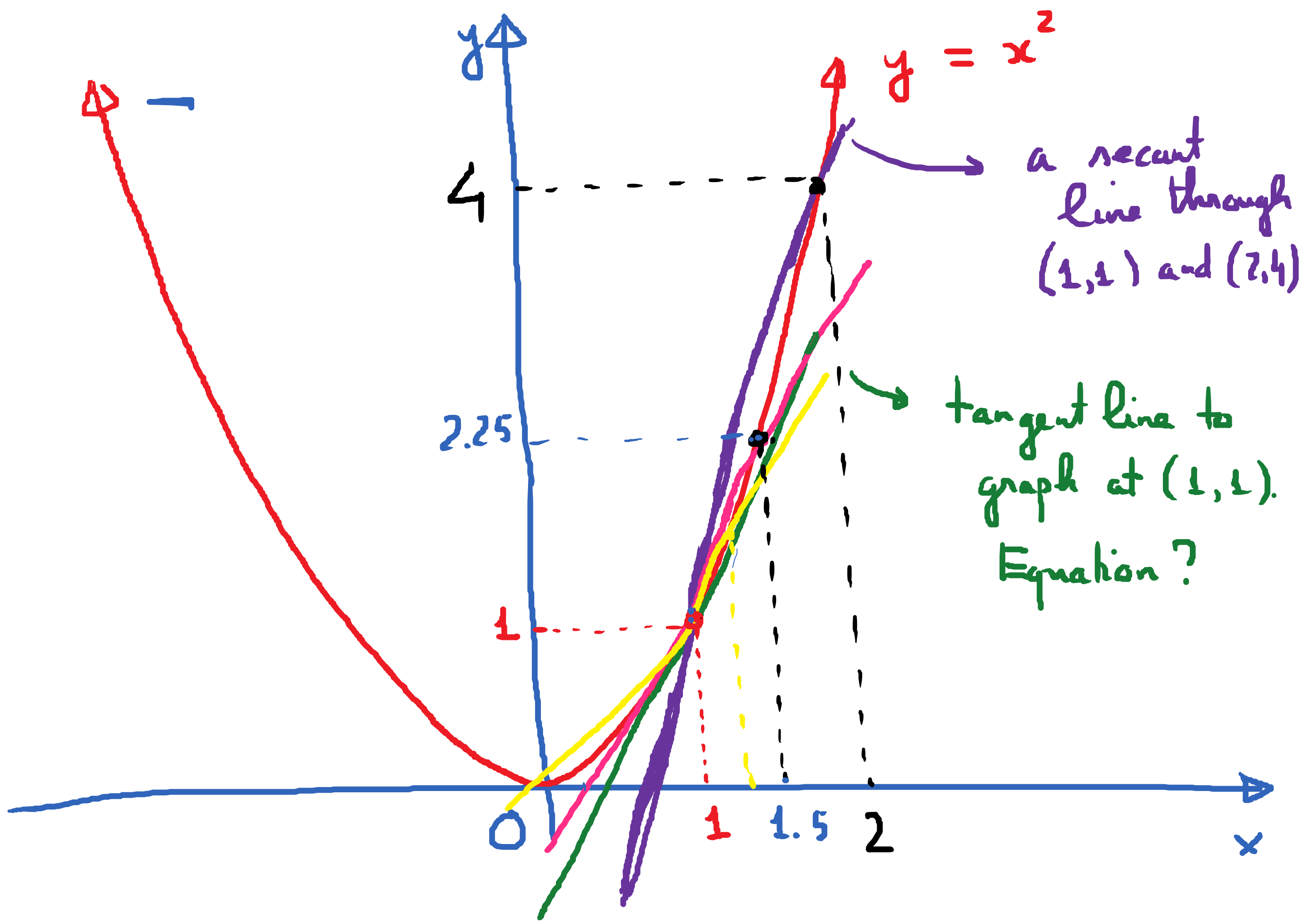
2.1-A Preview of Calculus

Monday, January 22, 2018 8:27 AM

1 The tangent line problem.

Problem: $f(x) = x^2$. Find the equation of the tangent line to the graph of f at the point $(1, 1)$.



Slope of the secant line through $(1,1)$ and $(2,4)$: $\dfrac{4-1}{2-1} = \dfrac{3}{1} = 3$

x	$y = x^2$	m_{sec} (slope of secant line through $(1,1)$ and (x,y))
2	4	3
1.5	2.25	$\dfrac{2.25-1}{1.5-1} = 2.5$
1.01	1.0201	$\dfrac{1.0201-1}{1.01-1} = 2.01$

(We let x get close to 1 from the right)

x	y	m_{sec}
0.5	0.25	$\dfrac{0.25-1}{0.5-1} = 1.5$
0.99	0.9801	$\dfrac{0.9801-1}{0.99-1} = 1.99$

(We let x get close to 1 from the left)

x	y	m_{sec} (slope of secant line that passes through $(1,1)$ and (b,b^2))
b	b^2	$\frac{b^2-1}{b-1} = m_{sec}$

Q: What does the expression $\boxed{\frac{b^2-1}{b-1}}$ get close to as the variable b gets closer and closer to 1?

From the numerical calculations:

* When b gets close to 1 from the right ($b = 2;\ 1.5,\ 1.01,\ 1.001 \ldots$)

  $\frac{b^2-1}{b-1}$ seems close to 2.

* When b gets close to 1 from the left ($b = 0.5;\ 0.9,\ 0.99,\ 0.999 \ldots$)

  $\frac{b^2-1}{b-1}$ seems close to 2 as well

To express the question what does $\frac{b^2-1}{b-1}$ get close to when b gets close to 1 from the right, we use:

$$\lim_{b \to 1^+} \frac{b^2-1}{b-1}$$

* from the left: $\lim_{b \to 1^-} \frac{b^2-1}{b-1}$

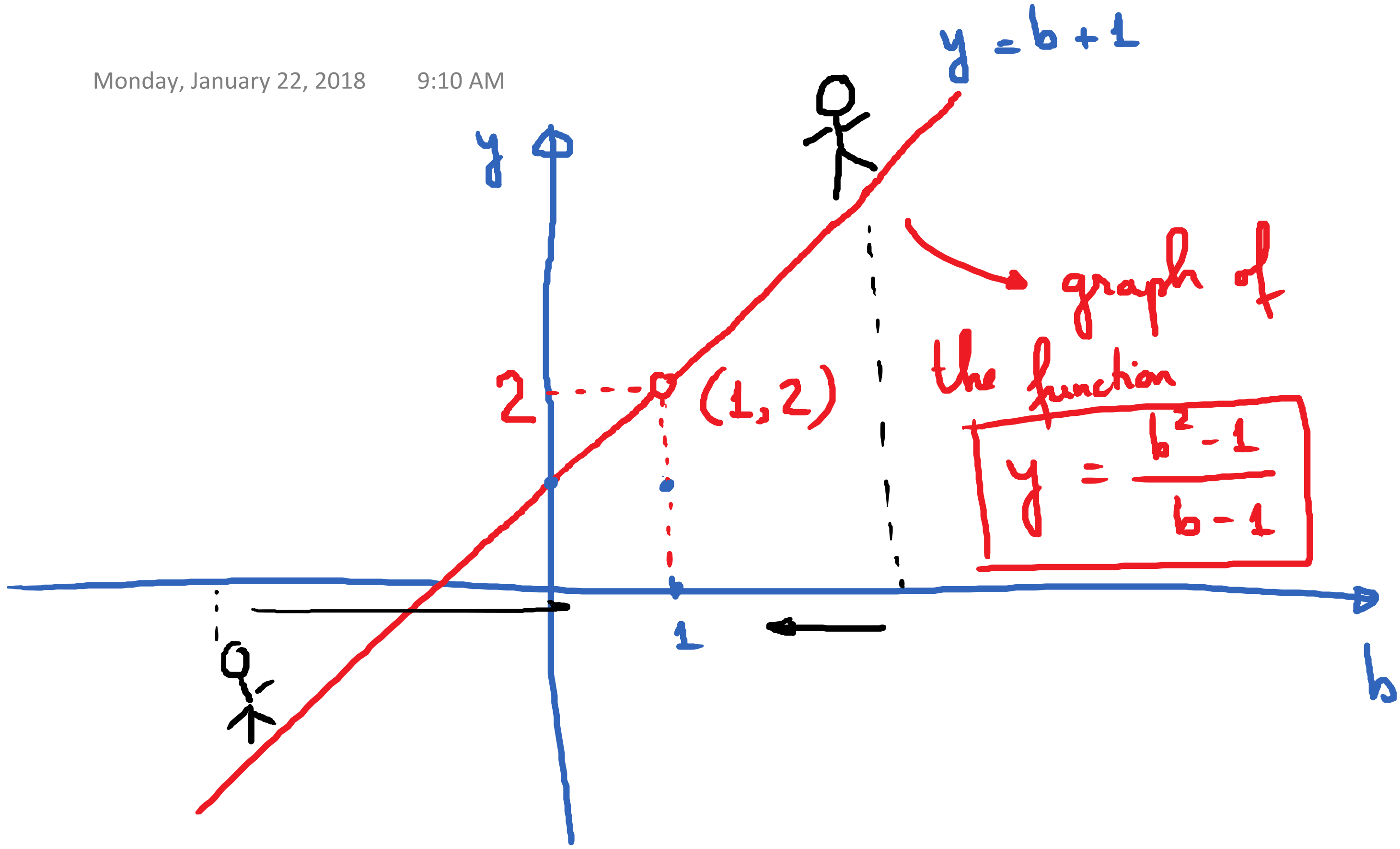
* Algebraic explanation of why m_{sec} approaches 2

(difference between squares.)

$$\frac{b^2-1}{b-1} = \frac{(b+1)\cancel{(b-1)}}{\cancel{b-1}} = b+1$$

if $b \neq 1$

$$\boxed{\frac{b^2-1}{b-1}} = \begin{cases} b+1 & \text{if } b \neq 1 \\ \text{undefined} & \text{if } b = 1 \end{cases}$$



Conclusion: $\displaystyle\lim_{b \to 1^+} \frac{b^2 - 1}{b - 1} = 2$

$$\lim_{b \to 1^-} \frac{b^2 - 1}{b - 1} = 2$$

$\rightarrow$ $\displaystyle\lim_{b \to 1} \frac{b^2 - 1}{b - 1} = \boxed{2}$ $\longrightarrow$ Slope of the tangent line to the graph of $y = x^2$ at $(1, 1)$

Equation: $y = 2x + b$.