Plug $x = 1$, $y = 1$ into equation:

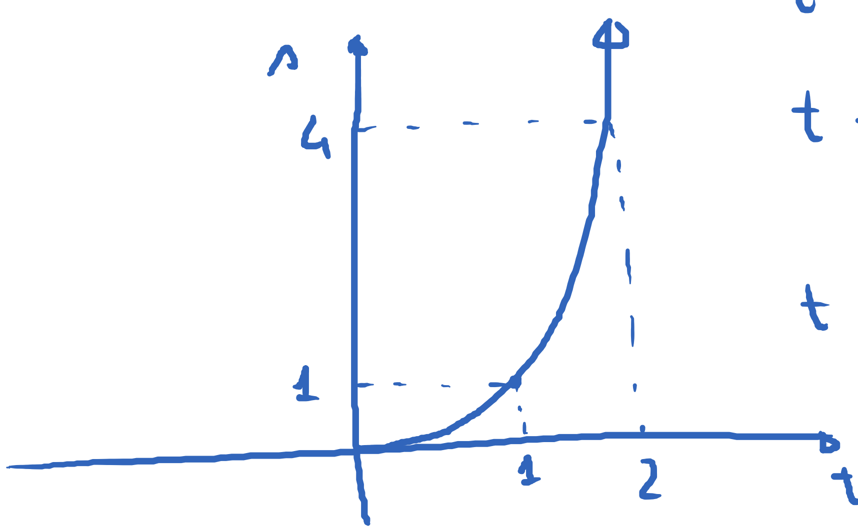
$$1 = 2 \cdot 1 + b. \quad \text{So, } b = -1.$$

Equation: $\boxed{y = 2x - 1}$

Who cares?

In physics, $s(t) = t^2$: position function of a moving object.

t is measured in seconds



$t = 1(s)$: object is 1 m away from

$t = 2(s)$: object is 4 m away from O.

Average speed of object from 1 s to 2 s:

$$\frac{4-1}{2-1} = 3 \ (m/s)$$

Instantaneous Speed at $t = 1$ (s) = Slope of the tangent line to the graph of the position function at $t = 1$.

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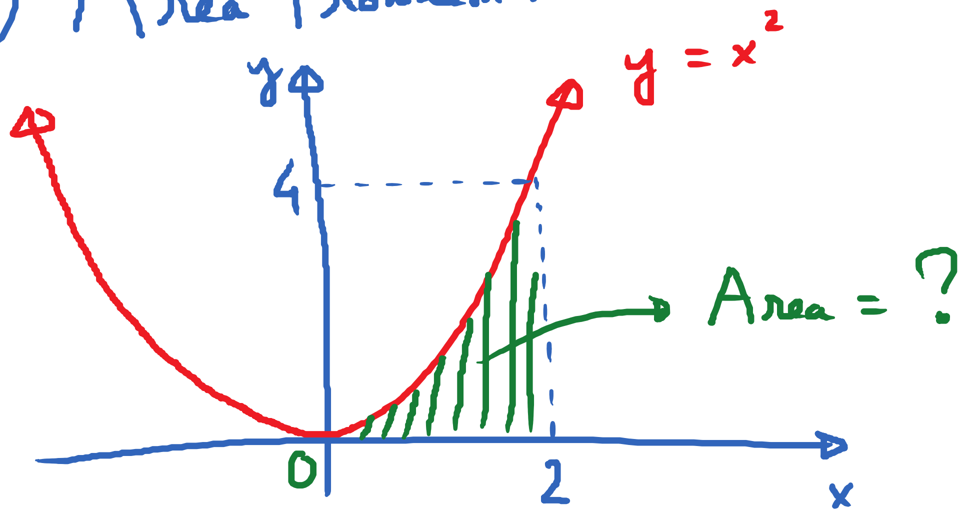
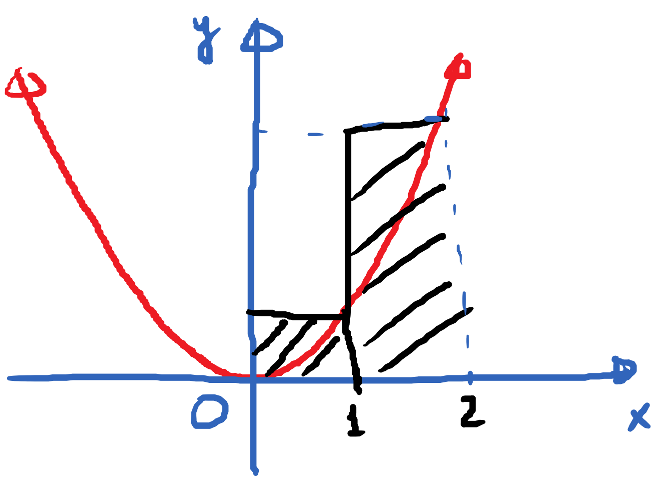
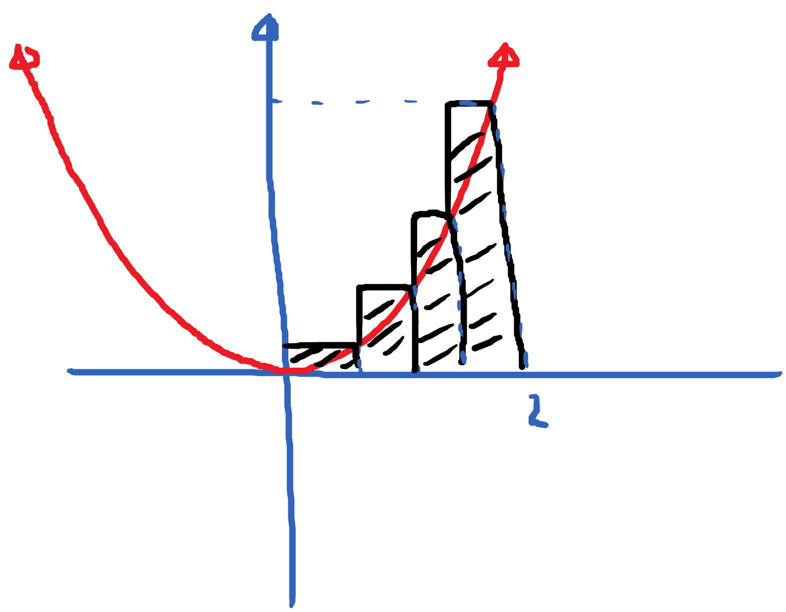
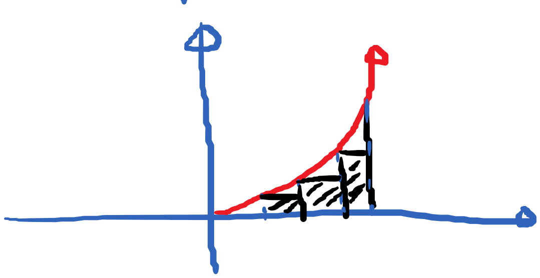
② Area Problem.



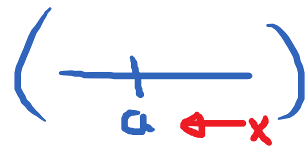


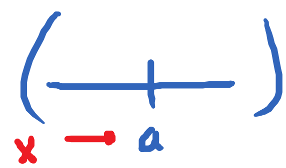




→ another limiting process.

Definition: $y = f(x)$ is a function.

We say that $\lim\limits_{x \to a^+} f(x) = b$ if the values of f get really close to b as the values of x get close to a from the right. 

Similarly, we say that $\lim\limits_{x \to a^-} f(x) = b$ if the values of f get really close to b as the values of x get close to a from the left 

Finally, we say that $\lim\limits_{x \to a} f(x) = b$ if

$$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = b.$$

Finding limits Numerically:

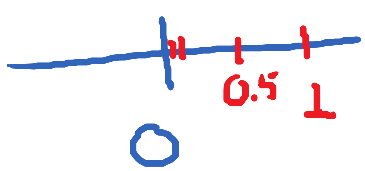
$$f(x) = \frac{\sin x}{x}.$$

Find $\lim_{x \to 0} \frac{\sin x}{x}$ numerically

x	$y = \frac{\sin x}{x}$
1	0.84
0.5	0.958
0.1	0.998
0.01	0.999

x	$y = \frac{\sin x}{x}$
−1	0.84
−0.5	0.958
−0.1	0.998
−0.01	0.999

$$\lim_{x \to 0^+} \frac{\sin x}{x} = 1 \qquad \lim_{x \to 0^-} \frac{\sin x}{x} = 1$$



$$\lim_{x \to 0} \frac{\sin x}{x} = 1.$$

* Find limit graphically.

Ex. $f(x) = \dfrac{1}{x}$.

Find $\lim\limits_{x \to 0^+} f(x)$.

$\lim\limits_{x \to 0^-} f(x)$

$\lim\limits_{x \to 0} f(x)$

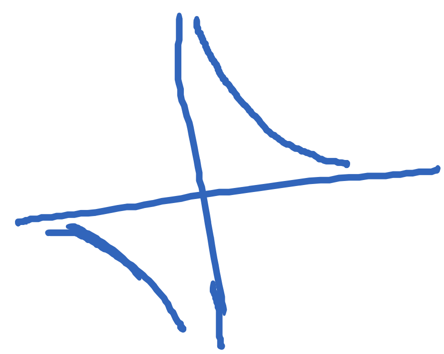
} Numerically.

$\lim\limits_{x \to 0^+} f(x)$

x	$f(x) = \frac{1}{x}$
1	1
0.5	2
0.1	10
0.01	100
0.001	1000
0.0001	10000
0.00001	100000
0.0000001	100000000

10 000 002

$\lim\limits_{x \to 0} \dfrac{1}{x} = \text{DNE}$



$\lim\limits_{x \to 0^+} \dfrac{1}{x} = \infty$; $\lim\limits_{x \to 0^-} \dfrac{1}{x} = -\infty$