2.2 and 2.3. Limit Laws

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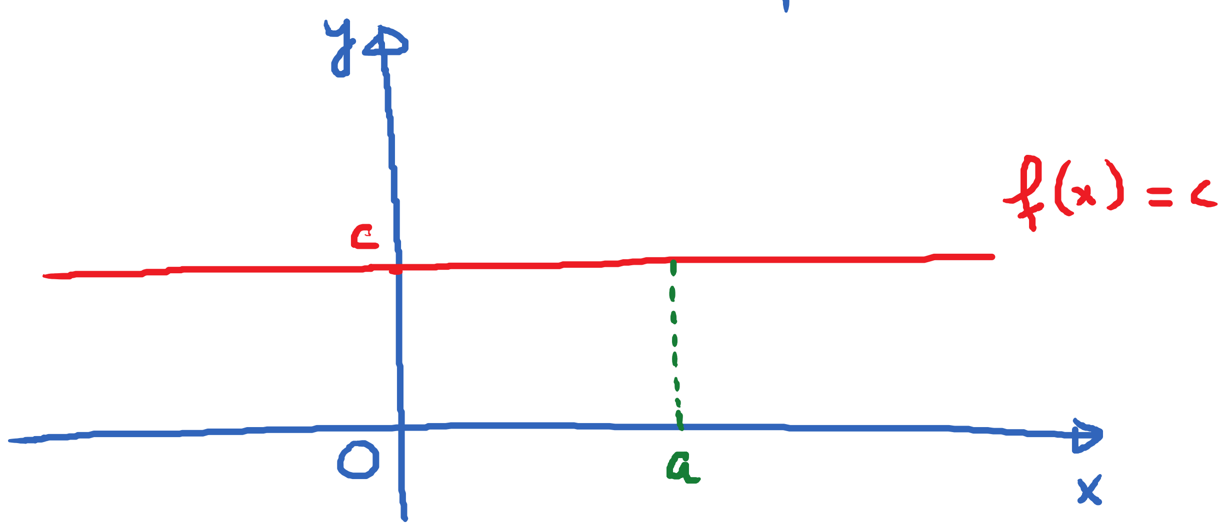
$\lim_{x \to a} f(x)$

- Numerically
  - $\lim_{x \to a^-} f(x)$
  - $\lim_{x \to a^+} f(x)$
- Graphically

Find limits Analytically.

* 2 basic limits:

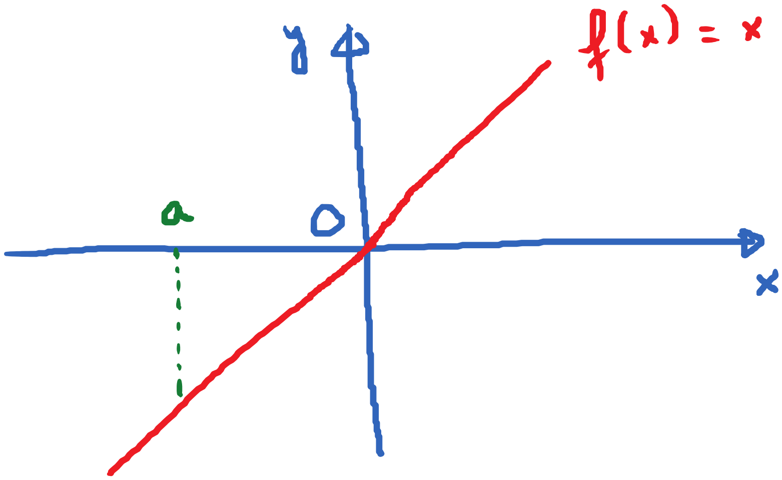
① c : constant. $\underbrace{f(x) = c}_{\text{constant function}}$ for all x.



$$\lim_{x \to a} f(x) = c.$$

In short, $\boxed{\lim_{x \to a} c = c}$

2 Identity function : $f(x) = x$



$$\lim_{x \to a} f(x) = a$$

In short, $\boxed{\lim_{x \to a} x = a}$

* limit Laws.

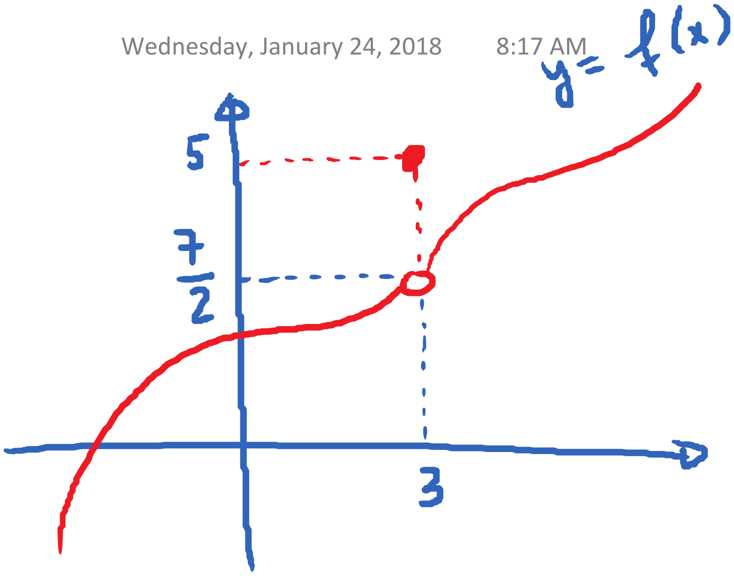
Given $\lim_{x \to a} f(x) = L$; $\lim_{x \to a} g(x) = M$.

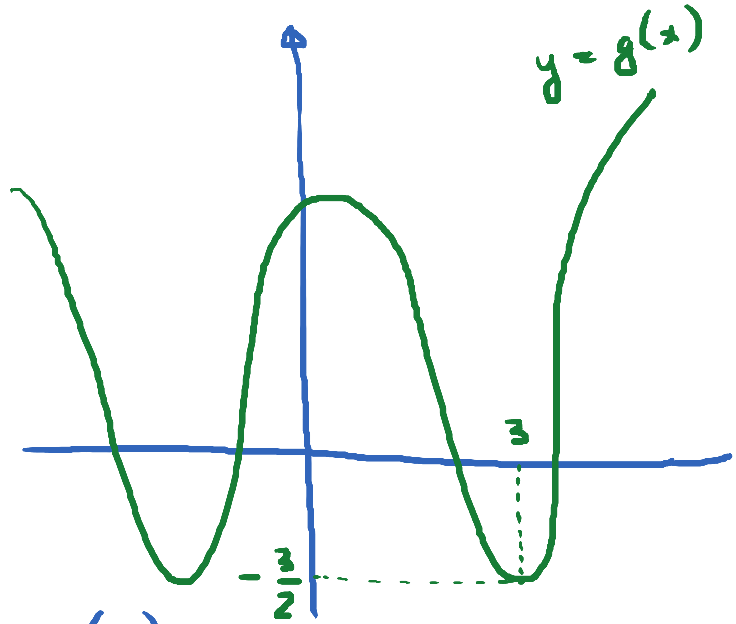
a <u>Sum Law:</u> $\lim_{x \to a} [f(x) + g(x)] = L + M$

or $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

b <u>Difference Law:</u> $\lim_{x \to a} [f(x) - g(x)] = L - M$

or $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$





$\lim_{x \to 3} h(x)$ and $\lim_{x \to 3} u(x)$

where $h(x) = f(x) + g(x)$. $u(x) = f(x) - g(x)$

$\lim_{x \to 3} h(x) = \frac{7}{2} + \left(-\frac{3}{2}\right) = 2$. $\lim_{x \to 3} u(x) = 5$

ⓒ Product Law: $\lim_{x \to a} [f(x) \cdot g(x)] = L \cdot M$

or $\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} [f(x)] \cdot \lim_{x \to a} [g(x)]$.

ⓓ Quotient Law: $\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right] = \frac{L}{M}$ (provided that $M \neq 0$)

or $\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ provided that $\lim_{x \to a} g(x) \neq 0$

(e) Constant Multiple Law:

Given $\lim_{x\to a} f(x) = L$

Ask: $\lim_{x\to a}\left(\frac{3}{2} f(x)\right) = \frac{3}{2}\cdot L$

Proof: By product law: $\lim_{x\to a}\left(\frac{3}{2} f(x)\right)$

$$= \underbrace{\left(\lim_{x\to a}\frac{3}{2}\right)}_{\frac{3}{2}}\cdot\left(\lim_{x\to a} f(x)\right)$$

1^{st} basic limit

$$= \frac{3}{2}\cdot L$$

In general, if c is any constant, then

$$\lim_{x\to a}\left(c f(x)\right) = cL$$

or

$$\lim_{x\to a}\left(c\cdot f(x)\right) = c\cdot \lim_{x\to a}\left(f(x)\right)$$

(f) Root Law: Given $\lim_{x\to a} f(x) = L$

$$\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{L}$$

or $\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$

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(g) Power Law:

$$\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$$

E.g. $\lim_{x \to -2} (x^3) = \left(\lim_{x \to -2} x\right)^3 = (-2)^3 = -8$

Power Law 2nd basic limit

Ex. Find these limits. List all the limit laws that apply when you find the limit

(1) $\lim_{x \to 0} (4x^2 - 2x + 3) = 3$ [Power, Sum, Diff, constant multiple, basic limits]

(2) $\lim_{x \to -2} \sqrt{x^2 - 6x + 3} = \sqrt{19}$ [Root, Power, Diff, Sum, constant mult., basic limits]

Bottom Line: When we try to find $\lim_{x \to a} f(x)$, the first thing to do is to plug $x = a$ into the function. If we get out a finite #, that is the answer. Done!