E.g. $f(x) = 3x^2 + x + 1$.

Verify that f is continuous on $(-\infty, \infty)$

This is a polynomial function. The Domain is $(-\infty, \infty)$. Hence, by Theorem above, f is continuous at every point in this domain.

E.g. $g(x) = \dfrac{x^2 - 1}{x - 1}$. Domain: $\{x \mid x \neq 1\}$

$(-\infty, 1) \cup (1, \infty)$

By Theorem above, g is continuous at every point in the interval $(-\infty, 1) \cup (1, \infty)$

* $\boxed{\text{Interval of continuity} = \text{Domain}}$

Ex. Find the intervals of continuity of the given function

1. $f(x) = \dfrac{5}{e^x - 2}$.
2. $g(x) = \dfrac{2}{x^2 + 5}$
3. $h(x) = \sqrt{\boxed{x^3 - x}}$.

Monday, January 29, 2018 9:21 AM

1. Find Domain: $e^x - 2 = 0$

$$e^x = 2$$

$$\ln e^x = \ln 2$$

$$x = \ln 2$$

Domain: $(-\infty, \ln 2) \cup (\ln 2, \infty)$

Interval of Continuity.

2. Domain of $g(x) = \frac{2}{x^2+5}$ is $(-\infty, \infty)$

3. $\sqrt{\text{Stuff}}$

Requirement: Stuff ≥ 0

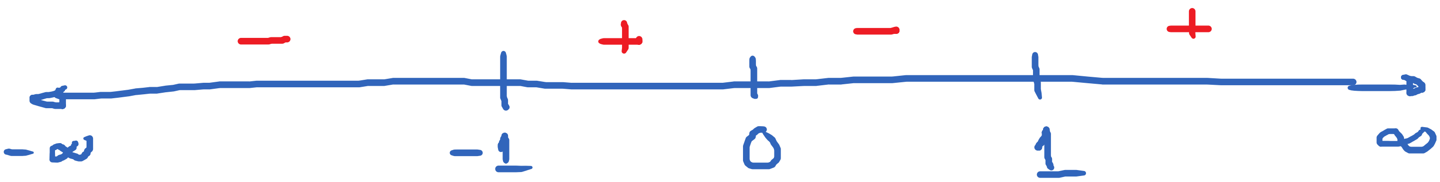
To find domain of our function:

$$\boxed{x^3 - x} \geq 0 .$$

$x^3 - x = 0$; $x(x^2-1) = 0$

$$x(x-1)(x+1) = 0$$

$$x = 0, \quad x = 1, \quad x = -1$$



Domain: $[-1, 0] \cup [1, \infty)$

Important problem solving technique:

Solve $f(x) \geq 0$ or $f(x) \leq 0$

$(f(x) > 0)$ or $(f(x) < 0)$

1st step: Find all values of x at which f is equal to zero or f is undefined.

2nd step: List these values on a number line. These values divide the number line into different intervals

3rd step: Choose a test point in each interval and plug it into $f(x)$. The sign of f at the test point tells us the sign of f on that whole interval. (Why?)

Theorem (Intermediate Value Theorem)

f: function

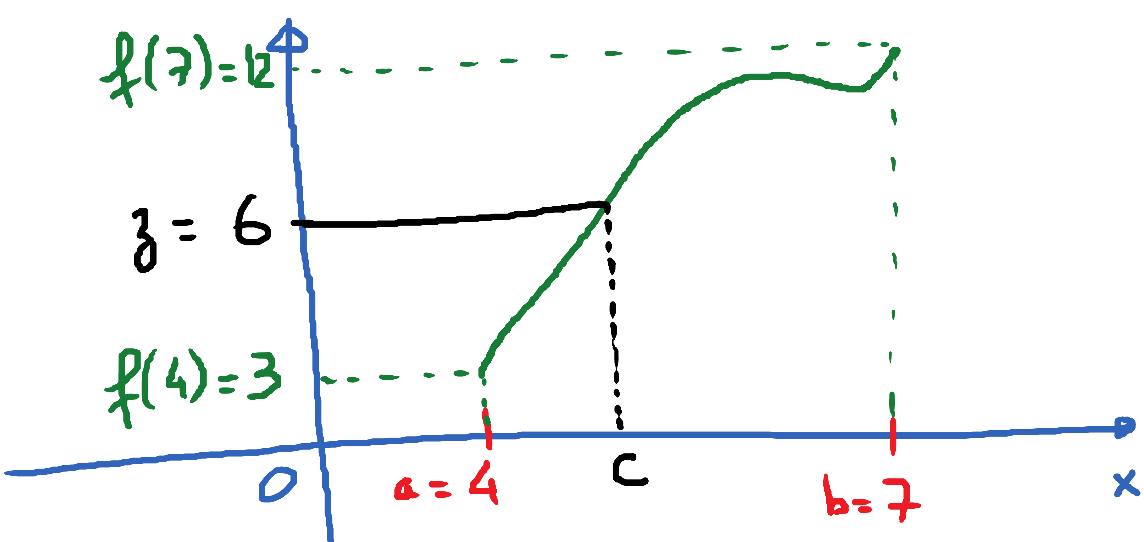
f is continuous on an interval $[a,b]$

z: any number in between $f(a)$ and $f(b)$

such that $f(a) < z < f(b)$

Conclusion: There exists a number c in $[a,b]$

such that $f(c) = z$.



Special case: if f is continuous on $[a,b]$ and $f(a) < 0$

and $f(b) > 0$

Conclusion: then there exists a number c in (a,b) such that $f(c) = 0$.

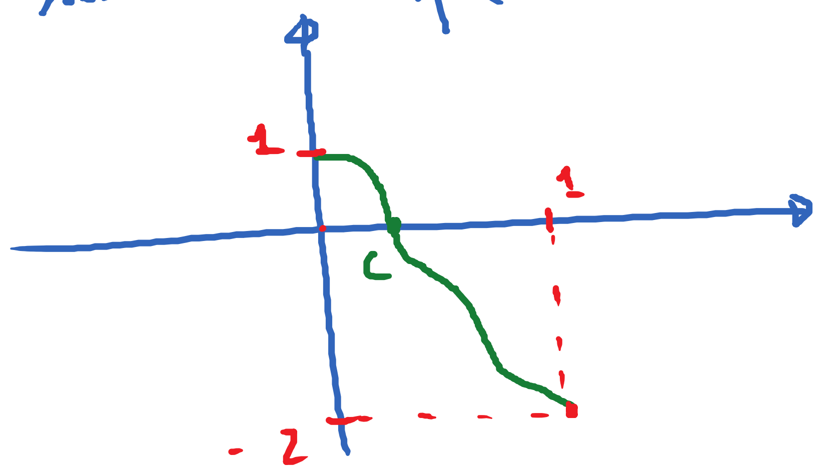
E.g. $f(x) = x^3 - x^2 - 3x + 1$.

f is continuous

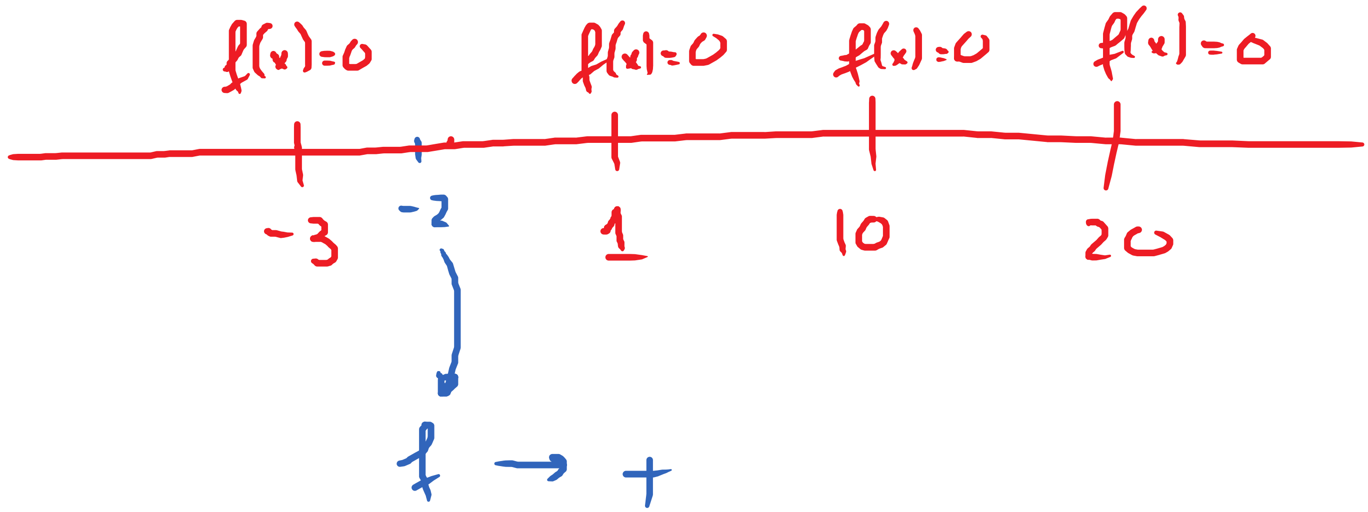
$f(0) = 1$; $f(1) = -2$

$f(0) > 0$; $f(1) < 0$

$\longrightarrow$ there will be a number c in $(0, 1)$ such that $f(c) = 0$



Back to the question:


f(x)=0
f(x)=0
f(x)=0
f(x)=0
-3
-2
1
10
20
f → +