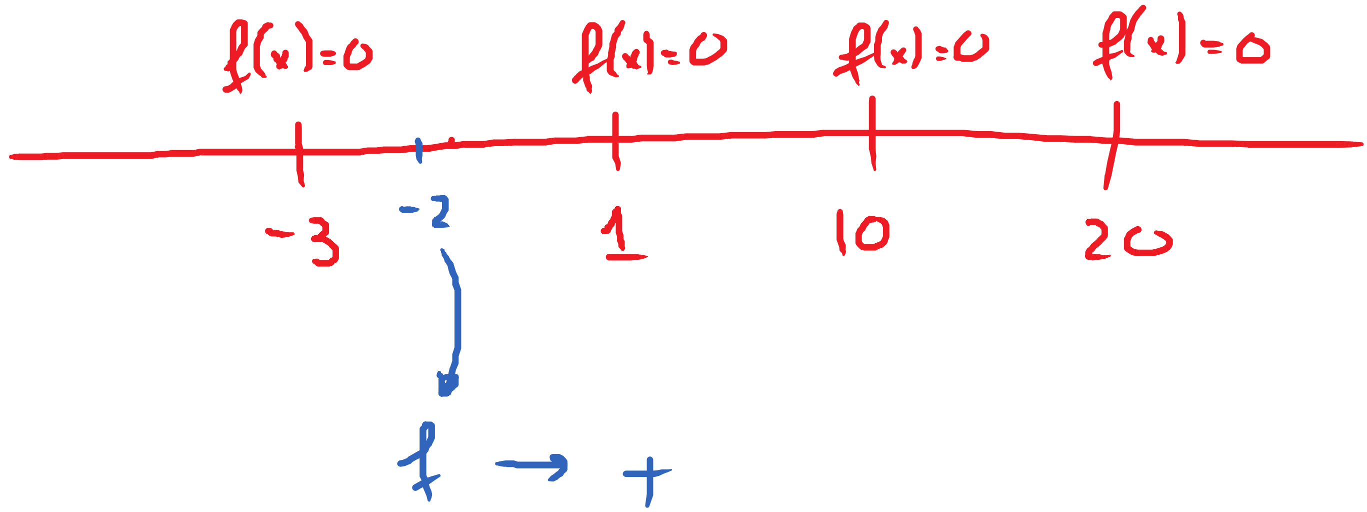
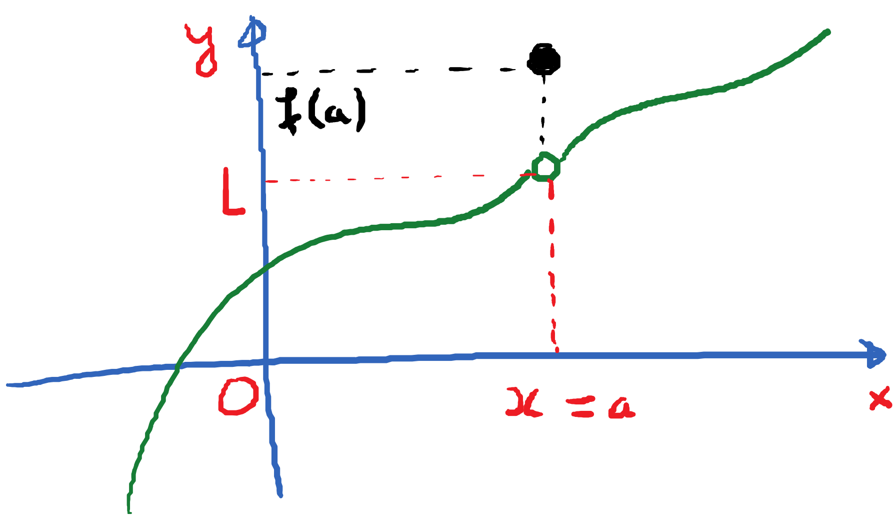


Types of Discontinuity

① Removable Discontinuity:



f has a removable discontinuity at $x = a$ if:

$\lim\limits_{x \to a} f(x)$ exists but

$\lim\limits_{x \to a} f(x) \neq f(a)$

E.g.

$$f(x) = \begin{cases} \boxed{\dfrac{x^2 + 3x + 2}{x + 2}} & \text{if } x \neq -2 \\ 3 & \text{if } x = -2 \end{cases}$$

Claim: f has a removable discontinuity at $x=-2$.

Why?

$$\lim_{x\to -2} f(x) = \lim_{x\to -2} \frac{x^2+3x+2}{x+2} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to -2} \frac{\cancel{(x+2)}(x+1)}{\cancel{x+2}} = \lim_{x\to -2} (x+1) = \boxed{-1}$$

$f(-2) = 3$

So, $\underbrace{\lim_{x\to -2} f(x)}_{-1} \neq \underbrace{f(-2)}_{3}$

$-1 \neq 3$

$\boxed{\text{Removable Discontinuity.}}$

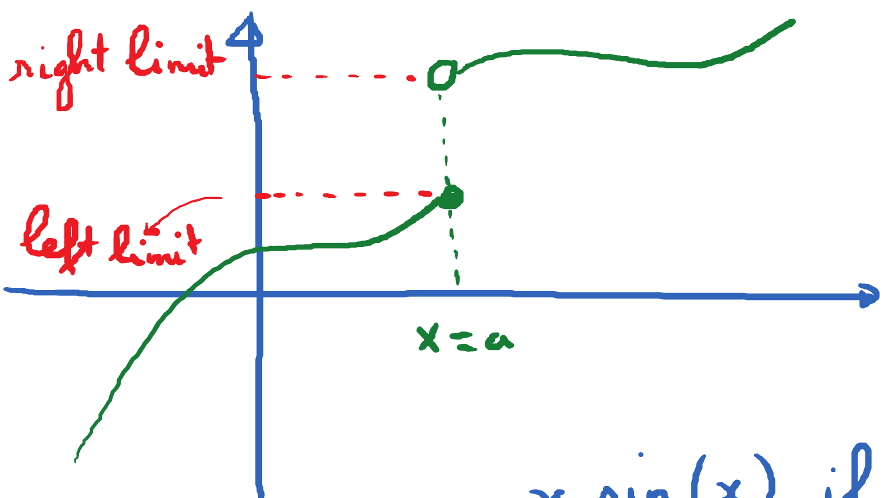
Redefine f as

$$\hat{f}(x) = \begin{cases} \dfrac{x^2+3x+2}{x+2} & \text{if } x \neq -2 \\ -1 & \text{if } x = -2 \end{cases}$$

(2) Jump Discontinuity.

f has a jump discontinuity at $x = a$ if

$$\boxed{\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)}$$



E.g. $f(x) = \begin{cases} x\sin(x) & \text{if } x < \pi \\ x\cos(x) & \text{if } x \geq \pi \end{cases}$

Claim: f has a jump discontinuity at $x = \pi$.

$$\lim_{x\to \pi^-} f(x) = \lim_{x\to\pi} x\sin(x) = \pi\cdot\sin(\pi) = \pi\cdot 0 = 0$$

$$\lim_{x\to \pi^+} f(x) = \lim_{x\to\pi} x\cdot\cos(x) = \pi\cdot\cos(\pi) = \pi\cdot(-1)$$
$$= -\pi$$

different

③ Infinite Discontinuity.

f has an infinite discontinuity at $x = a$ if

$$\lim_{x \to a^-} f(x) = -\infty \text{ or } \infty$$

or

$$\lim_{x \to a^+} f(x) = -\infty \text{ or } \infty$$

E.g. $f(x) = \dfrac{1}{x}$

Infinite Discontinuity

at $x = 0$