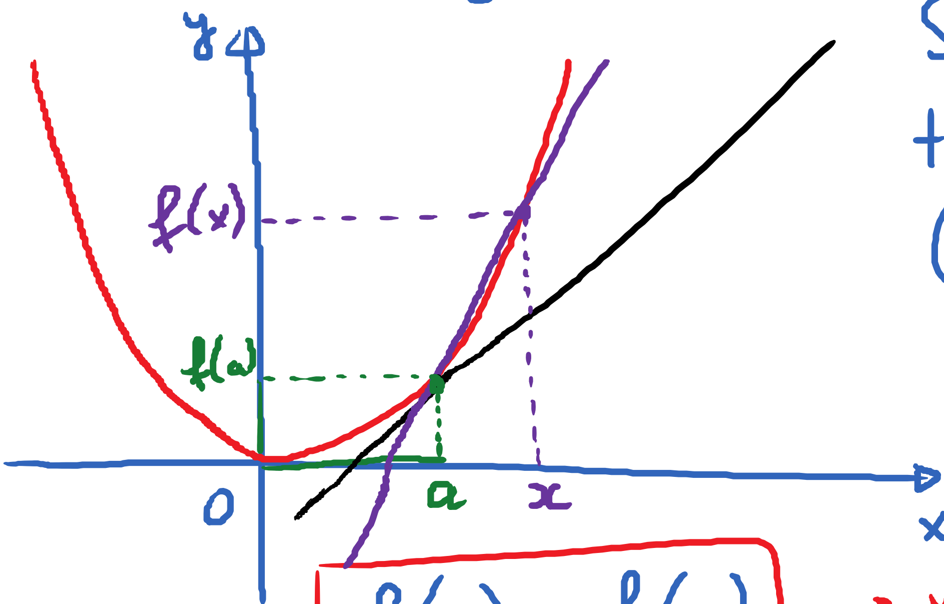
3.1. The Definition of the Derivative

Wednesday, January 31, 2018 8:25 AM

Back to the tangent line problem



Slope of secant line through $(a, f(a))$ and $(x, f(x))$

$$m_{sec} = \boxed{\frac{f(x) - f(a)}{x - a}}$$

what does this quantity get close to as x gets really close to a?

Slope of tangent line:

$$\boxed{m_{tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

provided that this limit exists

<u>E.g.</u> $f(x) = x^3$. $a = 2$.

Find the slope of the tangent line to the graph of f at $(2, 8)$.

$$m_{tan} = \lim_{x \to 2} \frac{x^3 - 8}{x - 2} \qquad \left(\frac{0}{0}\right)$$

$A^2 - B^2 = (A-B)(A+B)$ | $A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

$$
\begin{array}{r|l}
 & x^2 + 2x + 4 \\
\hline
\boxed{x} - 2 & \boxed{x^3} \qquad\qquad - 8 \\
 & -(x^3 - 2x^2) \\
 & \qquad \boxed{2x^2} \qquad - 8 \\
 & \qquad -(2x^2 - 4x) \\
 & \qquad\qquad \boxed{4x} - 8 \\
 & \qquad\qquad -(4x - 8) \\
\hline
 & \qquad\qquad\qquad 0
\end{array}
$$

$$\frac{x^3}{x} = x^2$$

$$\frac{2x^2}{x} = 2x$$

$$\frac{4x}{x} = 4$$

$$\frac{x^3 - 8}{x - 2} = x^2 + 2x + 4$$

$$\lim_{x \to 2} \frac{x^3 - 8}{x - 2} = \lim_{x \to 2} (x^2 + 2x + 4) = 4 + 4 + 4$$

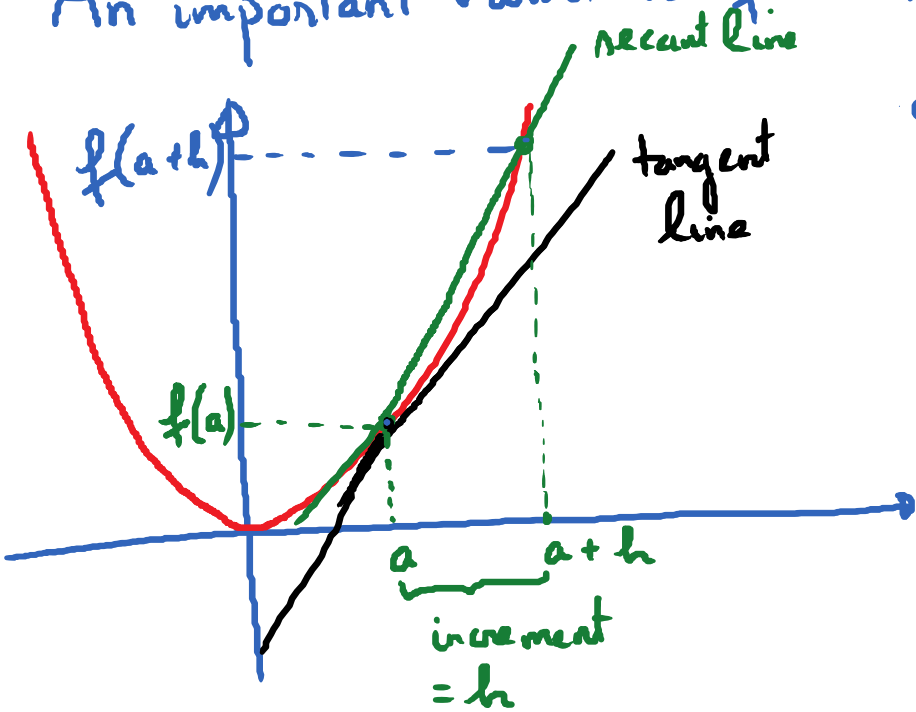
$$= \boxed{12}$$

$$\frac{x^3 - 8}{x - 2}$$

$$
\begin{array}{c|cccc}
2 & 1 & 0 & 0 & -8 \\
 & & 2 & 4 & 8 \\
\hline
 & \boxed{1} & \boxed{2} & \boxed{4} & \boxed{0}
\end{array}
$$

$$x^2 + 2x + 4$$

An important variation of the formula for m_{tan}



Slope of the secant line through $(a, f(a))$ $(a+h, f(a+h))$

$$m_{sec} = \frac{f(a+h) - f(a)}{\cancel{a}+h - \cancel{a}} = \boxed{\frac{f(a+h) - f(a)}{h}}$$

So, $$\boxed{m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

In Summary, the slope of the tangent line to the graph of $y = f(x)$ at the point $(a, f(a))$ can be calculated by:

$$\boxed{m_{tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

by:

$$\boxed{m_{\tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

Definition: The $\boxed{\text{derivative}}$ of a function $y = f(x)$ at the point $x = a$; denoted by $f'(a)$ (read as f prime of a) or $\left.\dfrac{dy}{dx}\right|_{x=a}$

(Leibnitz notation)

is defined as:

$$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

provided that the limit exists.

Note: If $f(x)$ is the position function of a moving object at time x, then $f'(a)$ = instantaneous velocity at time $x = a$.

If $f(x)$ is a velocity function, then

$f'(a)$ = acceleration at time $x = a$.

If $f(x)$ is a cost function, then

$f'(a)$ = marginal cost at production level $x = a$