E.g 1 Find $f'(3)$ for $f(x) = \frac{1}{x}$.

2 Find $f'(2)$ for $f(x) = \frac{1}{x^2}$

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$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3}{3} \cdot \frac{1}{3+h} - \frac{1}{3} \cdot \frac{3+h}{3+h}}{h} \quad \left(\frac{0}{0} \right)$$

$$= \lim_{h \to 0} \frac{\frac{3}{3(3+h)} - \frac{3+h}{3 \cdot (3+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3 - (3+h)}{3(3+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\cancel{3} \cancel{- 3} - h}{3(3+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{-h}{3(3+h)}}{\frac{h}{1}} = \lim_{h \to 0} \frac{-\cancel{h}}{3(3+h)} \cdot \frac{1}{\cancel{h}}$$

$$= \lim_{h \to 0} \frac{-1}{3(3+h)} = \boxed{-\frac{1}{9}}$$

(2) $f(x) = \dfrac{1}{x^2}$; $f'(2)$

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4}{4} \cdot \frac{1}{(2+h)^2} - \frac{1}{4} \cdot \frac{(2+h)^2}{(2+h)^2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4}{4(2+h)^2} - \frac{(2+h)^2}{4(2+h)^2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4 - (2+h)^2}{4(2+h)^2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4 - (4 + 4h + h^2)}{4(2+h)^2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\cancel{4} - \cancel{4} - 4h - h^2}{4(2+h)^2}}{h} = \lim_{h \to 0} \frac{\frac{-4h - h^2}{4(2+h)^2}}{\frac{h}{1}}$$

$$= \lim_{h \to 0} \frac{-4h - h^2}{4(2+h)^2 \cdot h} = \lim_{h \to 0} \frac{\cancel{h}(-4-h)}{4(2+h)^2 \cdot \cancel{h}} = \boxed{-\frac{1}{4}}$$

The derivative as a function.

The formula $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

gives us the derivative of the function $y = f(x)$ at a specific point $x = a$.

In most cases, we like to have a formula that can give us the derivative at any arbitrary point x. First, we just need to replace a by x in the formula:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(provided limit exists)

This defines a function of x. That function $y = f'(x)$ is called the derivative function of f.

In short, the derivative of f.

Domain of $f' = \{ x \mid f'(x) \text{ exists} \}$

E.g. $f(x) = x^3$.

$$f'(x) = \lim_{h \to 0} \frac{f(\boxed{x+h}) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} \quad \left(\frac{0}{0}\right)$$

Pascal Triangle.

$A+B$

$(A+B)^2$

1				
1	1			
1	2	1		
1	3	3	1	
1	4	6	4	1

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{- x^3}}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$

$$\lim_{h\to 0}\left(3x^2+3xh+h^2\right)=3x^2$$

$$\boxed{f'(x)=3x^2}$$

$$f'(100)=3\cdot 10000=30000$$

$$f'\left(\frac{1}{2}\right)=3\cdot\frac{1}{4}=\frac{3}{4}$$

E.g. Find $f'(x)$ where $f(x)=\sqrt{x}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\qquad\left(\frac{0}{0}\right)$$

$$=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$=\lim_{h\to 0}\frac{\cancel{x}+h\cancel{-x}}{h\cdot\left(\sqrt{x+h}+\sqrt{x}\right)}=\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}\cdot\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}.$$

If $f(x) = \sqrt{x}$, then $\boxed{f'(x) = \frac{1}{2\sqrt{x}}}$

$f'(100) = \frac{1}{20}$; $f'(81) = \frac{1}{18}$

$f'(0)$ DNE

$$\underbrace{f'(a)} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Slope of tangent line to $y = f(x)$ at $x = a$

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$