E.g.

$$f(x) = \begin{cases} \boxed{x \sin\left(\frac{1}{x}\right)} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

$$g(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Q: Does $f'(0)$ exist? Does $g'(0)$ exist? Why?

$$\boxed{f'(0)} = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h} \sin\left(\frac{1}{h}\right) - 0}{\cancel{h}} = \boxed{\underbrace{\lim_{h \to 0} \sin\left(\frac{1}{h}\right)}_{\text{DNE}}}$$

DNE

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(From graph limit $= 0$)

$$g'(0) = \boxed{\lim_{h \to 0} h \boxed{\sin\left(\frac{1}{h}\right)}}$$

$$-1 \leq \sin\left(\frac{1}{h}\right) \leq 1$$

Multiply both sides by h

$$-h \leq h\sin\left(\frac{1}{h}\right) \leq h$$

, By Squeeze Theorem

$$\boxed{\lim_{h \to 0} h\sin\left(\frac{1}{h}\right) = 0}$$

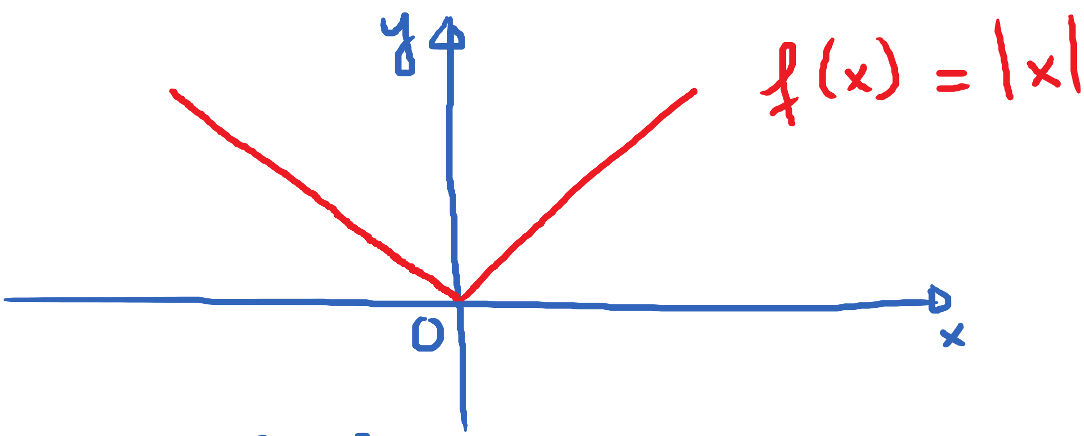
As $h \to 0$, 0

Therefore, $g'(0) = 0$.

Def: We say that f is differentiable at a point x if $f'(x)$ exists.

E.g. In the previous example, we saw that
f is NOT differentiable at 0
g is differentiable at 0.

E.g.



Consider $f'(0)$

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

DNE

$$= \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$= \boxed{\lim_{h \to 0} \frac{|h|}{h}}$$

DNE

$$\lim_{h \to 0^+} \frac{|h|}{h} = \lim_{h \to 0} \frac{h}{h} = \lim_{h \to 0} (1) = 1$$

$$\lim_{h \to 0^-} \frac{|h|}{h} = \lim_{h \to 0} \frac{(-h)}{h} = -1$$

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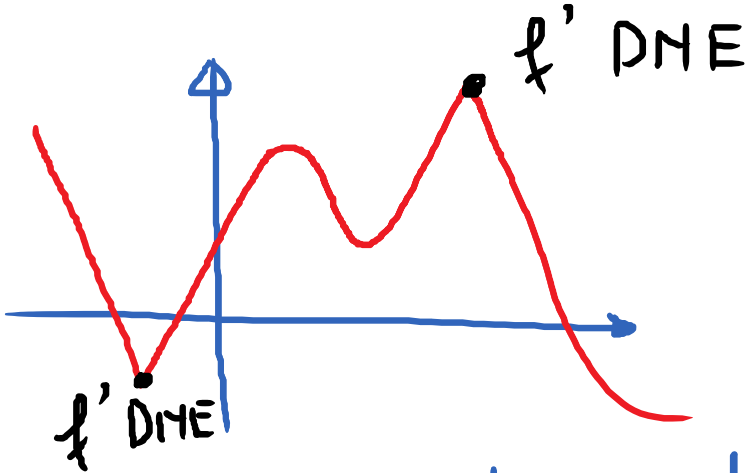
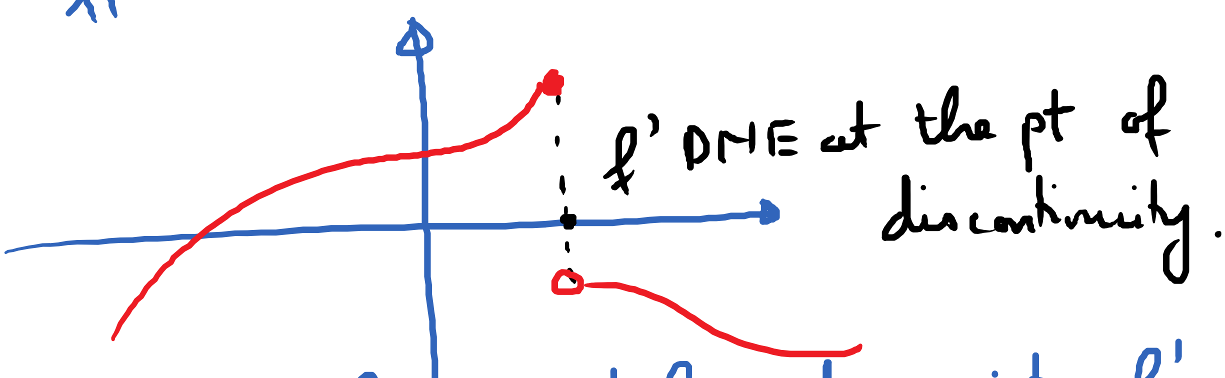
The function $f(x) = |x|$ is differentiable everywhere except for at $x = 0$.

In general, if a graph has a sharp corner at a point, the derivative doesn't exist there.

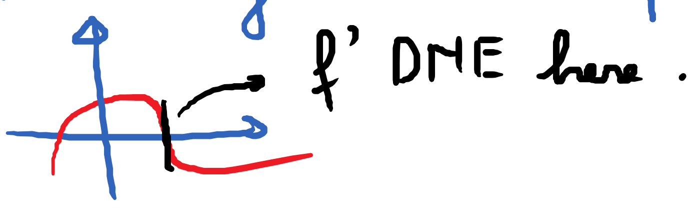
E.g.



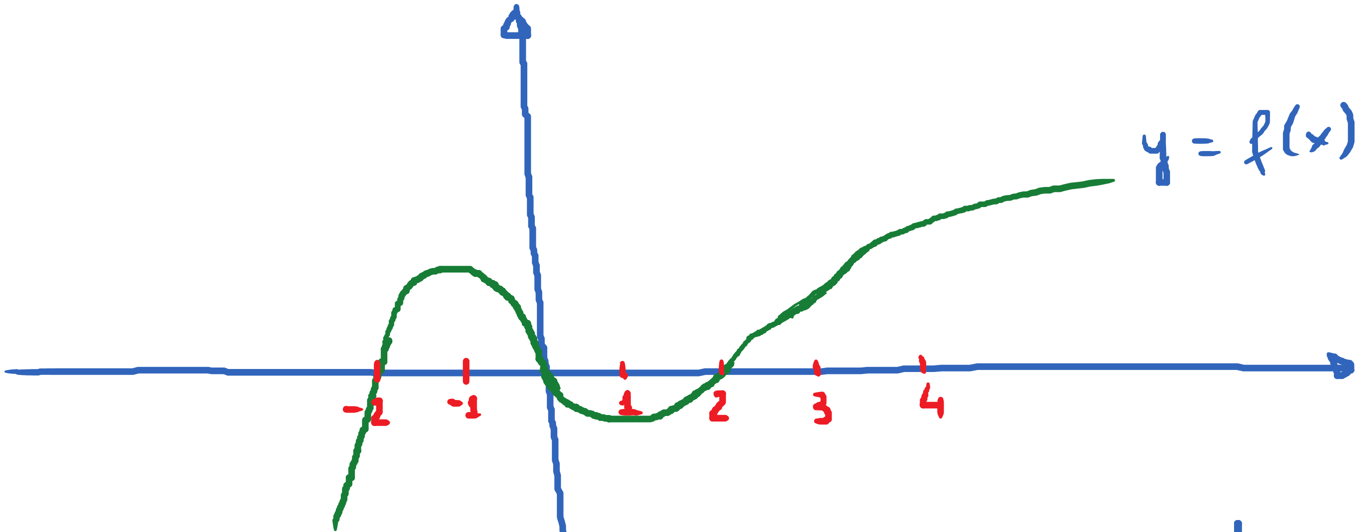
Also, if f is NOT continuous at a point, f is not differentiable there



If f has a vertical tangent line at a point, f' DNE there



E.x.



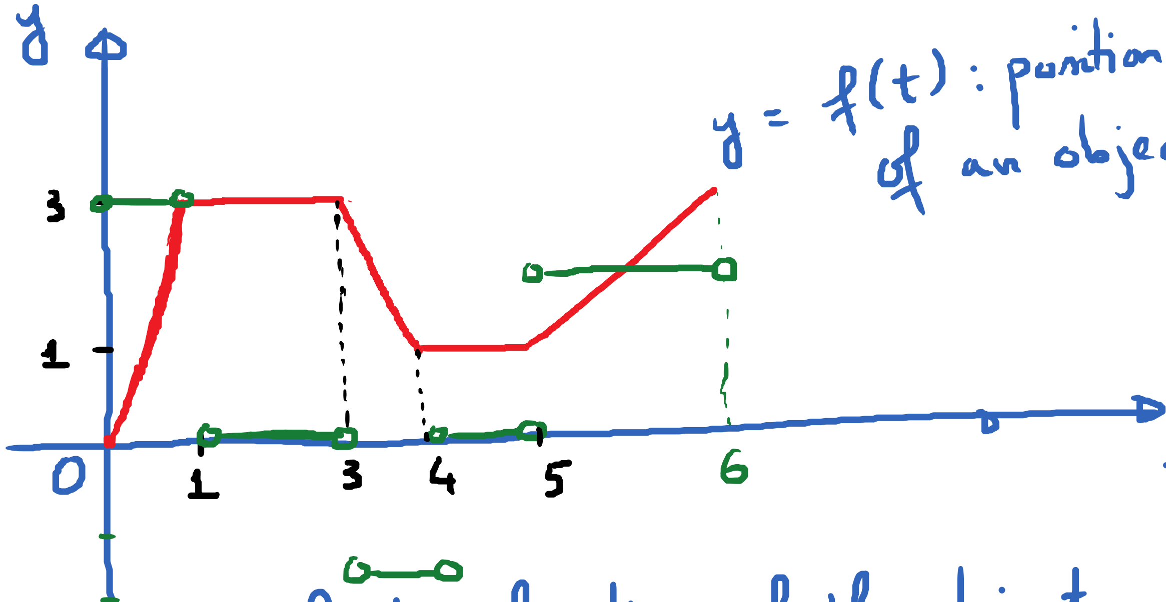
Arrange the following #s in increasing order:

$f'(-2), f'(0), f'(2), f'(4)$

$$f'(0) < f'(4) < f'(2) < f'(-2)$$

E.g.

$y = f(t)$: position function of an object.



Graph the velocity function of the object

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Differentiability is a stronger notion than continuity.

Not differentiable at those points.

Continuous everywhere