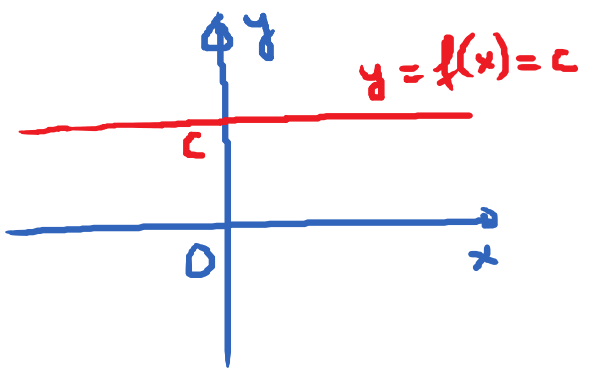
3.3. Differentiation Rules

1. Power Rule
2. Sum and Difference Rules
3. Product and Quotient Rules

---

* Derivative of a constant function.

If $f(x) = c$; c : constant, then $f'(x) = 0$



In Leibnitz notation:

$$\frac{d}{dx}[c] = 0$$

(1) Power Rule:

$$\frac{d}{dx}[x^2] = 2x. \quad \frac{d}{dx}[x^3] = 3x^2$$

In general, $\boxed{\frac{d}{dx}[x^n] = nx^{n-1}}$ n is a positive integer.

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$.

Monday, February 5, 2018 9:16 AM

If n is any real #, then

$$\frac{d}{dx}\left[x^n\right] = n \cdot x^{n-1}$$

E.g.

$$\frac{d}{dx}\left[x^{2018}\right] = 2018x^{2017}$$

$$\frac{d}{dx}\left[e^{2018}\right] = 0.$$

Power Rule

$$\frac{d}{dx}\left[\sqrt{x}\right] = \frac{d}{dx}\left[x^{\frac{1}{2}}\right] = \frac{1}{2} \cdot x^{\frac{1}{2}-1}$$

$$= \frac{1}{2} \cdot x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$\boxed{\frac{d}{dx}\left[\sqrt{x}\right] = \frac{1}{2\sqrt{x}}}$$

$$\frac{d}{dx}\left[\frac{1}{x}\right] = \frac{d}{dx}\left[x^{-1}\right] = -x^{-2} = \frac{-1}{x^2}.$$

$$\boxed{\frac{d}{dx}\left[\frac{1}{x}\right] = -\frac{1}{x^2}}$$

Ex. Find $\frac{d}{dx}\left[\frac{1}{\sqrt[4]{x^9}}\right]$

Ans: $-\frac{9}{4\sqrt[4]{x^{13}}}$

$$\frac{d}{dx}\left[x^{-9/4}\right] = -\frac{9}{4}x^{-\frac{13}{4}} = -\frac{9}{4x^{13/4}}$$

$$= -\frac{9}{4\sqrt[4]{x^{13}}} \qquad -\frac{9}{4\sqrt[4]{x^{12}\cdot x}}$$

$$= -\frac{9}{4x^3\cdot\sqrt[4]{x}}$$

Proof of the power rule.

Binomial Theorem

n choose 1 n choose 2

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \boxed{\binom{n}{3}}a^{n-3}b^3 + \dots + \boxed{\binom{n}{n-1}}ab^{n-1} + b^n$$

$(a+b)^0$	1				
$a+b$	1	1			
$(a+b)^2$	1	2	1		
$(a+b)^3$	1	3	3	1	
$(a+b)^4$	1	4	6	4	1

$$1\boxed{a^4} + 4\boxed{a^3 b} + 6\boxed{a^2 b^2} + 4\boxed{a b^3} + 1\boxed{b^4}$$

$$(a+b)^n = 1a^n + \boxed{?}a^{n-1}b + \boxed{?}a^{n-2}b^2 + \boxed{?}a^{n-3}b^3$$

$$+ \ldots \ldots \ldots +$$

$$\ldots + \boxed{?}a^2 b^{n-2} + \boxed{?}a b^{n-1} + 1b^n$$

$$\boxed{\binom{n}{k}} = \boxed{\frac{n!}{k!\,(n-k)!}}$$

$$5! = 5\cdot4\cdot3\cdot2\cdot1$$

$$\binom{3}{2} = \frac{3!}{2!\,1!} = \frac{3\cdot\cancel{2}\cdot\cancel{1}}{\cancel{2}\cdot\cancel{1}\cdot 1} = 3$$

$$(a+b)^2 = a^2 + \binom{2}{1}ab + b^2$$

$$= a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + \binom{3}{1}a^2b + \binom{3}{2}ab^2 + b^3$$

$$= a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^n = a^n + \underbrace{\boxed{\binom{n}{1}}}_{n}a^{n-1}b + \dots + \underbrace{\binom{n}{n-1}}_{n}ab^{n-1} + b^n$$

$$\binom{n}{1} = \frac{n!}{1!\,(n-1)!} = n$$

$$\binom{n}{n-1} = \frac{n!}{(n-1)!\,1!} = n$$

Monday, February 5, 2018 9:51 AM

$$(a+b)^n = a^n + na^{n-1}b + \text{Stuff} + nab^{n-1} + b^n$$

$f(x) = x^n$. Find the formula for $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\boxed{(x+h)^n} - x^n}{h} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{h\to 0} \frac{\cancel{x^n} + nx^{n-1}h + \text{Stuff} + nxh^{n-1} + h^n - \cancel{x^n}}{h}$$

$$= \lim_{h\to 0} \frac{nx^{n-1}h + \text{Stuff} + nxh^{n-1} + h^n}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}\left[nx^{n-1} + h\cdot\widetilde{\text{Stuff}}\right]}{\cancel{h}}$$

$$= \lim_{h\to 0} \left(nx^{n-1} + h\cdot\widetilde{\text{Stuff}}\right) = \boxed{nx^{n-1}}$$

Monday, February 5, 2018 9:59 AM

③ Sum, Difference and Constant Multiple Rule.

* Constant Multiple Rule:

$$\frac{d}{dx}\left[\sqrt{x}\right] = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}\left[\boxed{3}\sqrt{x}\right] = \frac{3}{2\sqrt{x}}$$

$$3 \cdot \frac{d}{dx}\left[\sqrt{x}\right]$$

In general, $\frac{d}{dx}\left[\boxed{c} f(x)\right] = c \cdot \frac{d}{dx}\left[f(x)\right]$

In prime notation: $\left(c \cdot f(x)\right)' = c \cdot f'(x)$

Sum Rule / Difference Rule.

E.g. $\left(\sqrt{x} \pm x^3\right)' = \frac{1}{2\sqrt{x}} \pm 3x^2$

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx}\left[f(x)\right] \pm \frac{d}{dx}\left[g(x)\right]$$

In prime notation, $\left(f(x) \pm g(x)\right)' = f'(x) \pm g'(x)$

E.g. Given $f(x) = x^3 - 4x^2 + 3x + 6$

Find $f'(x)$.

$$f'(x) = 3x^2 - 8x + 3$$

Find equation of tangent line to f at $x = 0$.

Slope $= f'(0) = 3$.

Point : $(0, f(0)) = (0, 6)$

Point - Slope Equation :

$$y - 6 = 3 \cdot (x - 0)$$

$$\boxed{y = 3x + 6}$$