Monday, February 5, 2018 10:12 AM

4 Product Rule:

$$\cancel{[f(x)\cdot g(x)]' = f'(x)\cdot g'(x)}$$

NOT CORRECT

E.g. $f(x) = x^2$; $g(x) = x^3$

$$\underbrace{[x^2\cdot x^3]'}_{\underbrace{[x^5]'}_{5x^4}} = \underbrace{\underbrace{(x^2)'}_{2x}\cdot\underbrace{(x^3)'}_{3x^2}}_{6x^3}$$

$$5x^4 \neq 6x^3$$

Correct Formula for Product rule:

$$\boxed{[f(x)\cdot g(x)]' = f'(x)\cdot g(x) + f(x)\cdot g'(x)}$$

In Leibnitz notation: u, v are functions of x

$$\boxed{\frac{d}{dx}[uv] = v\frac{du}{dx} + u\frac{dv}{dx}}$$

E.g. $w(x) = \underbrace{(2x^5 - 1)}_{f(x)} \cdot \underbrace{(x^2 + x)}_{g(x)}$

Find $w'(x)$.

By the product rule:

$$w'(x) = \underbrace{f'(x)}_{10x^4} \cdot g(x) + \underbrace{g'(x)}_{(2x+1)} \cdot f(x)$$

$$= 10x^4 \cdot (x^2 + x) + (2x+1) \cdot (2x^5 - 1)$$

$\longrightarrow$ expand and simplify...

* Quotient Rule:

u, v are functions of x

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}\left(\frac{\text{high}}{\text{low}}\right) = \frac{\text{low} \cdot d(\text{high}) - \text{high}\, d(\text{low})}{(\text{low})^2}$$

Prime notation:

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) \cdot g(x) - f(x) g'(x)}{(g(x))^2}$$

E.g. Find the derivative of $h(x) = \dfrac{3x+1}{4x-3}$.

Ans: $\dfrac{-13}{(4x-3)^2}$

$$h'(x) = \frac{3\cdot(4x-3) - (3x+1)\cdot 4}{(4x-3)^2}$$

$$= \frac{\cancel{12x} - 9 - \cancel{12}x - 4}{(4x-3)^2} = \frac{-13}{(4x-3)^2}$$

$h'(1) = \boxed{-13}$

Find equation of tangent to graph h when $x = 1$.

$y = -13x + 17$

$y = mx + b$; $y = -13x + b$

Plug $x = 1$ to $h(x)$: $h(1) = 4 \longrightarrow (1, 4)$

$4 = -13 + b \longrightarrow b = 17$

Or Point-Slope equation: $y - 4 = -13\cdot(x - 1)$

$\longrightarrow$ Simplify

HW #3:

$$f(x) = \frac{6x^3 - 7x + 1}{x^2}$$

$$= \frac{6x^3}{x^2} - \frac{7x}{x^2} + \frac{1}{x^2}$$

$$= 6x - \frac{7}{x} + \frac{1}{x^2}$$

$$f'(x) = 6 + \frac{7}{x^2} - \frac{2}{x^3}$$

$$\frac{d}{dx}\left[\frac{1}{x}\right] = -\frac{1}{x^2}$$

#7

$$h(x) = \boxed{x \cdot f(x)} + \boxed{5 \cdot g(x)}$$

$$h'(x) = 1 \cdot f(x) + x f'(x) + 5 \cdot g'(x)$$

$$h'(1) = f(1) + f'(1) + 5g'(1)$$

$$= 7 + (-1) + 30 = 36$$

#8: $h(x) = \frac{1}{x} + \frac{g(x)}{f(x)}$

$$h'(x) = -\frac{1}{x^2} + \frac{g'(x)\cdot f(x) - g(x)\cdot f'(x)}{(f(x))^2}$$

#10: $h'(x) = \frac{g(x)\cdot f'(x) - g'(x)\cdot f(x)}{(g(x))^2}$

$$h'(2) = \frac{g(2)\cdot f'(2) - g'(2)\cdot f(2)}{(g(2))^2}$$

$$= \frac{2\cdot(-1) - 1\cdot 2}{4} = \frac{-4}{4} = -1.$$

$h'(3)$ DNE.

$$h'(4) = \frac{g(4)\cdot f'(4) - g'(4)\cdot f(4)}{(g(4))^2}$$

$$= \frac{(2.5)\cdot 1 - 0\cdot f(4)}{(2.5)^2} = \frac{2.5}{(2.5)^2}$$

$$= \frac{1}{2.5} = \frac{2}{5}$$

Higher order derivative

$$f(x) = x^4$$

$$f'(x) = 4x^3$$

$$f''(x) = 12x^2 \leftarrow \text{second derivative}$$

$$f'''(x) = 24x \leftarrow \text{third derivative}$$

$$f^{(4)}(x) = 24 \leftarrow 4^{\text{th}} \text{ derivative}$$

$$f^{(5)}(x) = 0 \leftarrow 5^{\text{th}} \text{ derivative}$$

$$f^{(n)}(x) = 0 \text{ when } n \geqslant 5$$

In Leibnitz notation : $y = f(x)$

1^{st} derivative : $\dfrac{dy}{dx}$

2^{nd} derivative: $\dfrac{d^2y}{dx^2}$

3^{rd} derivative : $\dfrac{d^3y}{dx^3}$

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