3.5. Derivative of Trig Functions

Wednesday, February 7, 2018 9:29 AM

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left[u \pm v\right] = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$\frac{d}{dx}\left[k \cdot u\right] = k \cdot \frac{du}{dx} \quad (k : \text{constant})$$

$$\frac{d}{dx}\left[uv\right] = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

If $f(x) = \sin x$, then $f'(x) = \cos x$.

In Leibnitz notation: $\frac{d}{dx}[\sin x] = \cos x$.

$$\frac{d}{dx}[\cos x] = -\sin x$$

Find the formula for: $\frac{d}{dx}[\tan x]$

$$\frac{d}{dx}\left[\frac{\sin x}{\cos x}\right] = \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \frac{d}{dx}\left[\frac{1}{\cos x}\right]$$

$$= \frac{\cos x \cdot 0 - 1\cdot(-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$= \sec x \cdot \tan x$$

$$\frac{d}{dx}[\sec x] = \sec x \cdot \tan x$$

$$\frac{d}{dx}[\csc x] = -\csc x \cdot \cot x$$

$$\frac{d}{dx}[\sin x] = \cos x \ ; \quad \frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x \ ; \quad \frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x \ ; \quad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

Ex. $\boxed{y = \sin x}$

Find $\frac{dy}{dx}$; $\frac{d^2y}{dx^2}$; $\frac{d^3y}{dx^3}$; $\frac{d^4y}{dx^4}$; $\frac{d^5y}{dx^5}$

Use the pattern you observe to find:

$$\frac{d^{2018}y}{dx^{2018}}$$

$\frac{dy}{dx} = \cos x$; $\frac{d^2y}{dx^2} = -\sin x$; $\frac{d^3y}{dx^3} = -\cos x$

$\frac{d^4y}{dx^4} = \sin x$; $\frac{d^5y}{dx^5} = \cos x$.

To sum up: to find $\frac{d^ny}{dx^n}$ $(y = \sin x)$

- $\sin x$ if 4 divides evenly to n
- $\cos x$ if $R = 1$ $(4|n)$
- $-\sin x$ if $R = 2$ $(4|n)$
- $-\cos x$ if $R = 3$ $(4|n)$

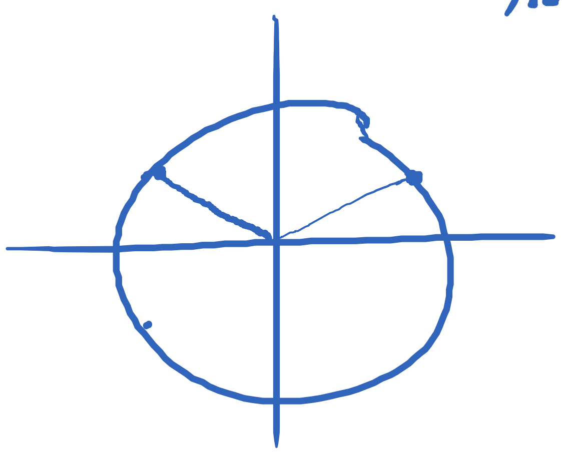
HW: Figure out the higher order derivatives for $y = \cos x$.

#7: $f'(x) = 1 + 2\sin x$

$1 + 2\sin x = 2 \quad (0 < x < 2\pi)$

$2\sin x = 1$

$\sin x = \frac{1}{2}$



$x = \frac{\pi}{6}, \frac{5\pi}{6}$