3.8. Implicit Differentiation

Wednesday, February 21, 2018 8:09 AM

Goal: To find derivatives implicitly.

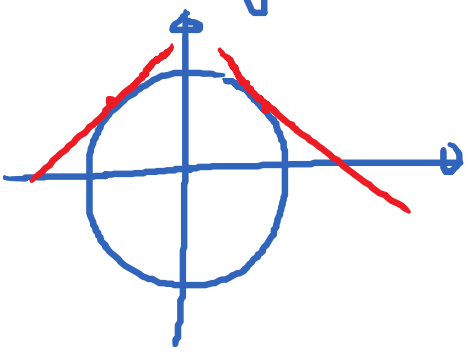
So far, y = formula in x (explicit)

E.g. $y = x^2 + 2x - 3$. → find y'.

→ Explicit Differentiation.

E.g. Equation: $\boxed{x^2 + y^2 = 25}$

→ y is given implicitly in terms of x.



How do we find $\frac{dy}{dx}$?

Take the derivative with respect to x of both sides:

$$\frac{d}{dx}\left(x^2 + y^2\right) = \frac{d}{dx}(25)$$

$\frac{d}{dx}\left(x^2 + y^2\right)$ → Sum Rule

$\frac{d}{dx}(25) = 0$ → 25 is a constant

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = 0$$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

→ To find $\frac{dy}{dx}$; get $\frac{dy}{dx}$ by itself:

$$2y \cdot \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{2x}{2y}$$

$$\boxed{\frac{dy}{dx} = -\frac{x}{y}}$$

Find the slope of the tangent line to the graph of $x^2 + y^2 = 25$ at $(3,4)$?

Slope at $(3,4)$: $\boxed{-\frac{3}{4}}$

Equation of tangent line:

Point-Slope Equation:

$$y - 4 = -\frac{3}{4}(x - 3)$$

$$y - 4 = -\frac{3}{4}x + \frac{9}{4}$$

$$y = -\frac{3}{4}x + \frac{9}{4} + 4$$

$$\boxed{y = -\frac{3}{4}x + \frac{25}{4}}$$

E.x. $4x^5 + \tan y = y^2 + 5x$

Find $\frac{dy}{dx}$?

E.x. $x^3y + xy^3 = -8$

Find $\frac{dy}{dx}$?

Solved in class.

E.g. Find the equation of the tangent line to the curve

$$x^2 + y^2 = \left(2x^2 + 2y^2 - x\right)^2$$

at $\left(0, \frac{1}{2}\right)$.

$$\frac{d}{dx}\left(x^2 + y^2\right) = \frac{d}{dx}\left[\left(2x^2 + 2y^2 - x\right)^2\right]$$

$$2x + 2y\frac{dy}{dx} = 2\left(2x^2 + 2y^2 - x\right)\cdot\left(4x + 4y\frac{dy}{dx} - 1\right)$$

Plug $x = 0$; $y = \frac{1}{2}$ to the above equation:

$$\frac{dy}{dx} = 2\cdot\left(\frac{1}{2}\right)\cdot\left(2\frac{dy}{dx} - 1\right)$$

$$\frac{dy}{dx} = 2\frac{dy}{dx} - 1 \rightarrow \boxed{\frac{dy}{dx} = 1} \rightarrow \text{slope}$$

Pt-Slope Eq:

$$y - \frac{1}{2} = x$$

$$\boxed{y = x + \frac{1}{2}}$$

E.g. Given $x^2 + y^2 = 25$.

We found $\frac{dy}{dx}$. $\frac{dy}{dx} = -\frac{x}{y}$.

Q: Find $\frac{d^2y}{dx^2}$.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(-\frac{x}{y}\right)$$

$$= -\frac{y \cdot 1 - x \cdot \frac{dy}{dx}}{y^2}$$

$$= -\frac{y - x \cdot \left(-\frac{x}{y}\right)}{y^2}$$

$$= -\frac{y + \frac{x^2}{y}}{y^2} = \boxed{-\frac{y^2 + x^2}{y^3}}$$