3.9. Derivatives of Exponential and Log Functions

Monday, February 26, 2018 8:15 AM

$$\frac{d}{dx}(x^n) = n x^{n-1}$$

$$\frac{d}{dx}(u^n) = n u^{n-1} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\cos u) = -\cos u \frac{du}{dx}$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\tan u) = \sec^2 u \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arcsin u) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\arctan u) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

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1) Derivatives of Exponential Functions

If $f(x) = e^x$, then $\boxed{f'(x) = e^x}$

In Leibnitz notation: $\boxed{\frac{d}{dx}(e^x) = e^x}$

If u is a function of x, then

$$\boxed{\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx}}$$

In Newton's notation:

$$\boxed{\left(e^{f(x)}\right)' = e^{f(x)} \cdot f'(x)}$$

Ex. Find the derivatives:

(a) $\underbrace{\frac{d}{dx}(e^{\pi})}_{0}$

(b) $\underbrace{\frac{d}{dx}(e^{-x})}_{-e^{-x}}$

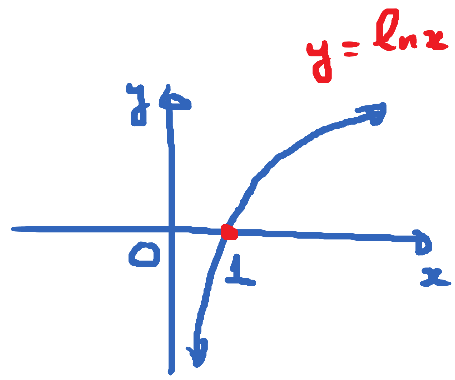
(c) $\underbrace{\frac{d}{dx}(e^{\sec x})}_{e^{\sec x} \cdot \sec x \cdot \tan x}$

(d) $\frac{d}{dx}(x \cdot e^{2x}) = e^{2x} + 2xe^{2x}$

$= e^{2x}(1 + 2x)$

2 Derivatives of Log Functions:

Natural log function: $y = \ln x$



($\ln 1 = 0$; $\ln e = 1$; $\ln e^2 = 2$

$\ln e^x = x$)

Claim: $\boxed{\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}}$; $\boxed{\dfrac{d}{dx}(\ln u) = \dfrac{1}{u} \cdot \dfrac{du}{dx} \quad \left(= \dfrac{u'}{u} \right).}$

Goal: See why $\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$?

$y = \ln x \iff \boxed{e^y = x}$

Use implicit differentiation to take $\dfrac{dy}{dx}$ of this

$\dfrac{d}{dx}(e^y) = \dfrac{d}{dx}(x) \longrightarrow e^y \cdot \boxed{\dfrac{dy}{dx}} = 1$

So, $\dfrac{dy}{dx} = \dfrac{1}{e^y} = \dfrac{1}{x}$.

We just proved that: $\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$.

Find the derivative of $y = a^x$ when a may not be e.

For e.g. Find $\underbrace{\dfrac{d}{dx}(2^x)}$? $\dfrac{d}{dx}(\pi^x)$?

$$\boxed{y = 2^x}$$

Hint #1: Rewrite this in terms of base e.

Hint #2: take $\ln$ of both sides & do implicit differentiation.

$2 = e^{\ln 2}$

$2^x = \left(e^{\ln 2}\right)^x$

$y = e^{x \ln 2}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\overset{u}{\boxed{x \ln 2}}}\right)$$

$$= e^{x \ln 2} \cdot (\ln 2) = 2^x (\ln 2)$$

$$\frac{d}{dx}\left(2^x\right) = 2^x \cdot \ln 2$$

$$\rightarrow \frac{d}{dx}\left(\pi^x\right) = \pi^x \cdot \ln \pi .$$

$$\rightarrow \boxed{\frac{d}{dx}\left(a^x\right) = a^x \cdot \ln a .}$$ (a is a positive constant)

Using Hint #2 to find $\frac{dy}{dx}$ where $y = 2^x$.

$\boxed{y = 2^x} \rightarrow$ take ln of both sides

$\ln y = \ln(2^x) \rightarrow \ln y = x \cdot \ln 2$

$\rightarrow$ Implicitly differentiate both sides w.r.t. x.

$$\rightarrow \frac{d}{dx}(\ln y) = \frac{d}{dx}(x \cdot \ln 2)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \ln 2 \rightarrow \frac{dy}{dx} = y \ln 2 = \boxed{2^x \ln 2}$$

Now: Find derivatives of $y = \log_a x$ where a may not be e.

E.g. Find $\boxed{\frac{d}{dx}(\log_2 x)}$ or $\boxed{\frac{d}{dx}(\log_{10} x)}$

written as $\log x$

$$\left(\log_2 8 = 3 \; ; \; \log_{10}(1{,}000{,}000) = 6 \quad \log_5\left(\frac{1}{25}\right) = -2 \right)$$

Find $\frac{d}{dx}(\log_2 x)$. (Hint: let $y = \log_2 x$. Find $\frac{dy}{dx}$)

$$y = \log_2 x \longleftrightarrow 2^y = x$$

Take $\frac{d}{dx}$ of both sides: $\frac{d}{dx}(2^y) = \frac{d}{dx}(x)$

$$2^y \ln 2 \cdot \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{2^y \ln 2} = \boxed{\frac{1}{x \ln 2}}$$