$$\frac{d}{dx}(\log x) = \frac{1}{x\ln(10)}$$

$$\frac{d}{dx}(\log_{\pi} x) = \frac{1}{x\ln(\pi)}$$

$\longrightarrow$ In general, $\boxed{\frac{d}{dx}(\log_a x) = \frac{1}{x\ln(a)}}$

$a > 0$ ; $a \neq 1$

Summary:

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(a^x) = a^x \cdot \ln a$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x\ln a}$$

$u$: function of $x$

$$\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(a^u) = a^u \ln a \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\log_a u) = \frac{1}{u\ln a} \cdot \frac{du}{dx}$$

Useful properties of the natural log function:

1. $\ln(uv) = \ln(u) + \ln(v)$
2. $\ln\left(\frac{u}{v}\right) = \ln(u) - \ln(v)$
3. $\ln(u^{p}) = p \ln u.$

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E.g. $\frac{d}{dx}\left(\ln\left[(2x+1)(3x+5)\right]\right)$

$= \frac{d}{dx}\left[\ln(2x+1) + \ln(3x+5)\right]$ (Use property 1 of ln)

$= \frac{d}{dx}\left(\ln(2x+1)\right) + \frac{d}{dx}\left(\ln(3x+5)\right)$

$= \frac{2}{2x+1} + \frac{3}{3x+5}$

E.g. $\frac{d}{dx}\left[\boxed{\ln(3x+2)^5}\right]$

$= \frac{d}{dx}\left[5\ln(3x+2)\right]$ (Use property ③ of ln)

$= 5 \cdot \frac{d}{dx}\left[\ln(3x+2)\right] = 5 \cdot \frac{3}{3x+2} = \frac{15}{3x+2}$.

Ex. Find the derivative

① $\frac{d}{dx}\left(\ln(\sin x)\right)$

② $\frac{d}{dx}\left(\ln\left(\sqrt{2x^2+7}\right)\right)$

③ $\frac{d}{dx}\left(\ln\left(\frac{(2x+1)^3}{\sqrt{x^2+3}}\right)\right)$

Solved this in class

Find the derivative of functions of the form

$$y = f(x)^{g(x)}$$

E.g. $y = x^x$. Find $\dfrac{dy}{dx}$?

$$\ln y = \ln\left(x^x\right)$$

$$\ln y = x \ln x \quad \rightarrow \text{property of } \ln$$

Implicitly differentiate both sides w.r.t. $x$.

→ product rule

$$\frac{1}{y} \cdot \frac{dy}{dx} = \ln x + x \cdot \frac{1}{x}.$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \ln x + 1$$

So, $\dfrac{dy}{dx} = y(\ln x + 1) = x^x(\ln x + 1)$.

Another way to solve this:

$$y = x^x = \left(e^{\ln x}\right)^x = e^{x \ln x}$$

$$y = e^u \longrightarrow \frac{dy}{dx} = e^u \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = e^{x \ln x} \cdot \frac{d}{dx}(x \ln x) = e^{x \ln x}(\ln x + 1)$$

$$= x^x(\ln x + 1)$$