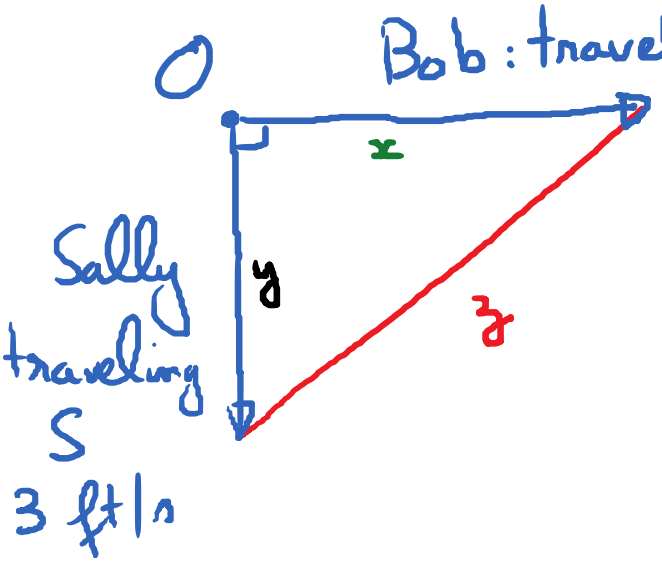
4.1. Related Rates

Wednesday, February 21, 2018 9:14 AM



Q: 10 second later

What is the rate of change of the distance between them?

x : distance from Bob to O. $\frac{dx}{dt} = 4$ ft/s

y : distance from Sally to O. $\frac{dy}{dt} = 3$ ft/s

z : distance between them

Q: Find $\frac{dz}{dt}$?

By Pythagorean Theorem:

$$x^2 + y^2 = z^2$$

→ Take derivative of both sides with respect to t

$$\frac{d}{dt}\left(x^2 + y^2\right) = \frac{d}{dt}\left(z^2\right)$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$x = 40$, $\frac{dx}{dt} = 4$, $y = 30$, $\frac{dy}{dt} = 3$, $\frac{dz}{dt} = ?$, $z = \sqrt{(40)^2 + (30)^2} = 50$

$$40 \cdot 4 + 30 \cdot 3 = 50 \cdot \frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{250}{50} = 5 \text{ ft/s}$$

Related Rates Problems

1. 2 or more quantities involved. All the quantities are changing with respect to time.
2. These quantities are related by an equation or a set of equations

(3) Know (given) the rate of change w.r.t. time of all but one quantity.

(4) Find the rate of change of the remaining quantity.

Approach:

(1) Identify the changing quantities.

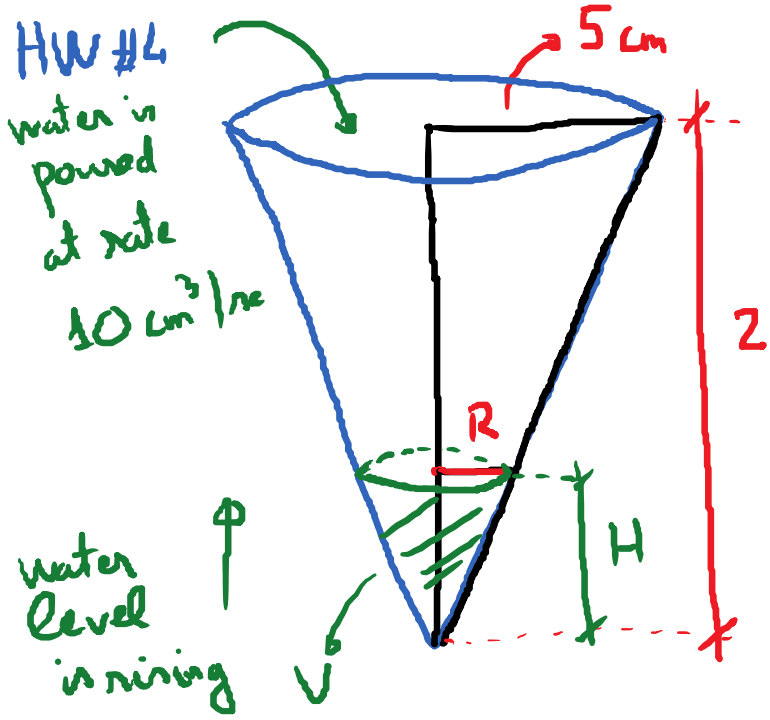
(2) Name them using variables such as $x, y, z, \ldots$
→ Keep in mind that all these are functions of t.

(3) Use geometry, etc. to find an equation or set of equations that relate them.

(4) Differentiate the equation with respect to t.

(5) Plug in the given information.

HW #4



Changing quantities:

Volume of water : V

Radius: R

Height of water: H

$\frac{dV}{dt} = 10\ cm^3/s$

Find $\frac{dH}{dt} = ?$

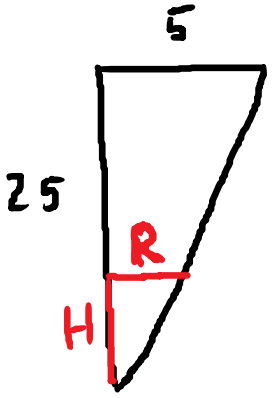
$$V = \frac{\pi R^2 \cdot H}{3}$$

→ Take $\frac{d}{dt}$.

$$\frac{dV}{dt} = \frac{\pi}{3}\frac{d}{dt}\left(R^2 \cdot H\right)$$

$$\frac{dV}{dt} = \frac{\pi}{3}\cdot\left(2R\frac{dR}{dt}H + R^2\frac{dH}{dt}\right)$$

10 2 ?



$\frac{5}{25} = \frac{R}{H}$ → $R = \frac{1}{5}H$

When $H = 2$, $R = \frac{2}{5}$.

$\frac{dR}{dt} = \frac{1}{5}\frac{dH}{dt}$

$$10 = \frac{\pi}{3} \cdot \left(2 \cdot \frac{2}{5} \cdot \frac{1}{5} \frac{dH}{dt} \cdot 2 + \left(\frac{2}{5} \right)^2 \left(\frac{dH}{dt} \right) \right)$$

$$10 = \frac{\pi}{3} \left(\frac{8}{25} \frac{dH}{dt} + \frac{4}{25} \frac{dH}{dt} \right)$$

$$10 = \frac{\pi}{3} \cdot \left(\frac{12}{25} \right) \frac{dH}{dt} \longrightarrow \frac{dH}{dt} = \ldots$$