4.10. Anti Derivatives.

Wednesday, April 4, 2018 8:53 AM

Definition of an antiderivative of a function:

f: function defined on an interval I.

A function F is an antiderivative of f on I

if $F'(x) = f(x)$ for every x in I.

In short, an antiderivative of a function is another function whose derivative is equal to the given function.

E.g. $f(x) = x$; defined on $(-\infty, \infty)$

Find an antiderivative of this:

$F(x) = \frac{1}{2}x^2$ (Check: $\left(\frac{1}{2}x^2\right)' = \frac{1}{2} \cdot 2x = x$)

$$G(x) = \frac{1}{2}x^2 + 1$$

$$H(x) = \frac{1}{2}x^2 + \pi$$

Any function that looks like:

$$F(x) = \frac{1}{2}x^2 + C \ ; \ C: \text{constant}$$

will be a solution.

E.g. $f(x) = x^2$

$F(x) = \frac{1}{3}x^3$ is an antiderivative.

In general, any antiderivative of $f(x) = x^2$ must have the form:

$$\boxed{F(x) = \frac{1}{3}x^3 + C = \frac{x^3}{3} + C \qquad C: \text{constant}}$$

The formula

$\frac{x^3}{3} + C$ is called $\boxed{\text{the general antiderivative}}$ of x^2

$f(x) = x^{2018}$

The general antiderivative of f: $\frac{x^{2019}}{2019} + C$.

$f(x) = x^n$; n is a constant; $n \neq -1$

The general antiderivative of f: $\frac{x^{n+1}}{n+1} + C$

---

when $n = -1$; $f(x) = \frac{1}{x}$

The general antiderivative of f : $\ln|x| + C$

Find the general antiderivative of $f(x) = \sqrt{x} = x^{\frac{1}{2}}$

$$F(x) = \boxed{\frac{2x^{3/2}}{3} + C}$$

(Check solution: $\left(\frac{2x^{3/2}}{3} + C\right)' = \frac{2}{\cancel{3}} \cdot \frac{\cancel{3}}{2} \cdot x^{\frac{1}{2}}$)

Important Notation: notation for the general antiderivative.

$\int f(x)\,dx$ = the general antiderivative of $f(x)$

(people also call this the indefinite integral of f)

$$\underbrace{\int f(x)\,dx}_{\text{read as: antiderivative of } f} = F(x) + C \quad \text{s.t. } F'(x) = f(x)$$

Table of very useful Antiderivatives

Function $f(x)$	General Antiderivative
$f(x) = x^n$; $n \neq -1$	$\boxed{\int x^n\,dx} = \dfrac{x^{n+1}}{n+1} + C$ $(n \neq -1)$
$f(x) = \dfrac{1}{x}$	$\int \dfrac{1}{x}\,dx = \ln\lvert x\rvert + C$
$f(x) = e^x$	$\int e^x\,dx = e^x + C$
$f(x) = \cos x$	$\int \cos x\,dx = \sin x + C$
$f(x) = \sin x$	$\int \sin x\,dx = -\cos x + C$
$f(x) = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$

$f(x) = \csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$
$f(x) = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$f(x) = \csc x \cot x$	$\int \csc x \cot x\,dx = -\csc x + C$
$f(x) = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}\,dx = \arctan x + C$
$f(x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$
$f(x) = k$, k : constant	$\int k\,dx = kx + C$

Wednesday, April 4, 2018 9:44 AM

Useful Properties of Antiderivatives / Indefinite Integrals.

$$\boxed{\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx}$$

Warning:

$$\cancel{\int f(x)g(x)\,dx = \int f(x)\,dx \cdot \int g(x)\,dx}$$

We don't have this.

$$\boxed{\int k \cdot f(x)\,dx = k \cdot \int f(x)\,dx}$$

Ex. Find the antiderivatives.

1. $\int \left(7x^{2/5} + 8x^{-4/5}\right) dx$

2. $\int \left(2\sin x - \sec^2 x\right) dx$

3. $\int e^2 \, dx$

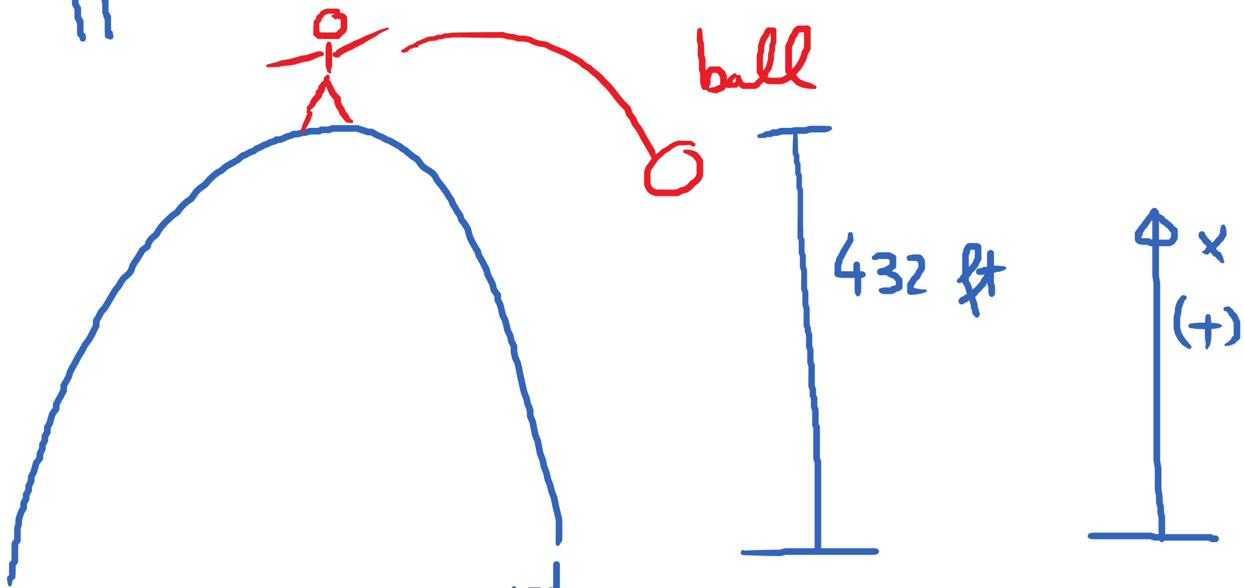
4. $\int \left(4\sqrt{x} - \sqrt[4]{x}\right) dx$

5. $\int (x+1)(2x-1) \, dx$

6. $\int \frac{2x^5 - \sqrt{x}}{x} \, dx$

* 7. $\int \frac{2 + x^2}{1 + x^2} \, dx$

An application:



Ball is thrown upward with an initial speed $v_0 = 48$ ft/s.

from top of a cliff 432 ft above ground.

Constant acceleration due to gravity is

-32 ft/s^2.

Q: Find position function $s(t)$ of ball.

$$a(t) = -32 \longrightarrow v(t) = -32t + C$$

Since $v(0) = C \longrightarrow C =$ initial speed $= 48$ ft/s.