4.3. Finding Maxima and Minima of a function.

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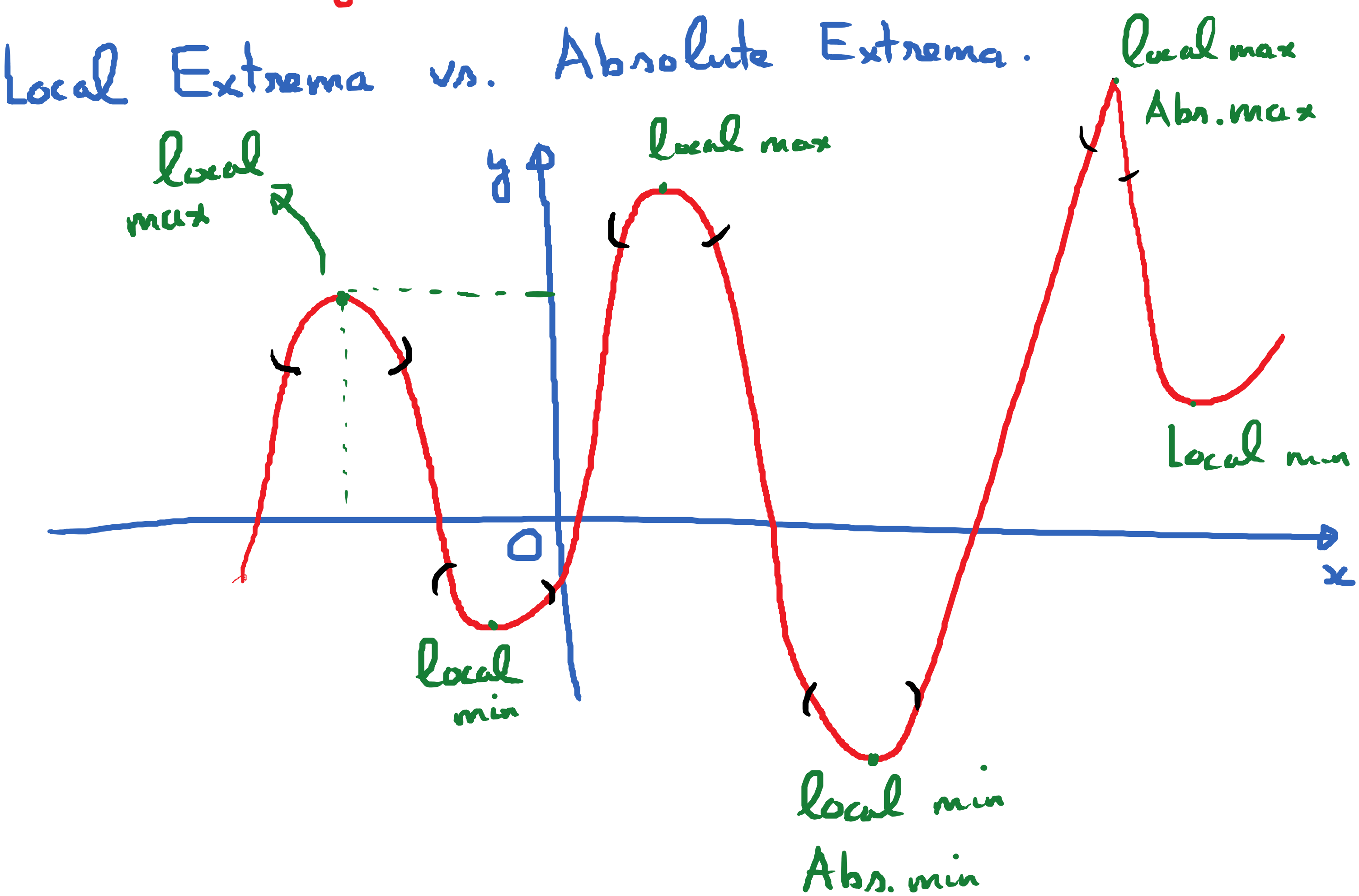
Local Extrema vs. Absolute Extrema.



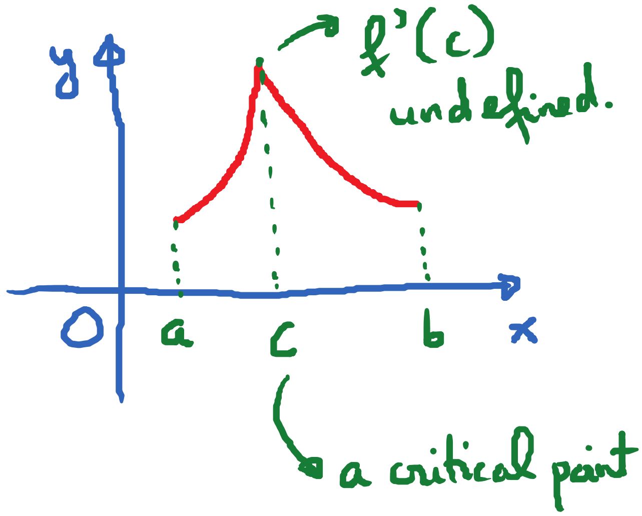
① Technique to find absolute max and absolute min of a function over a closed interval.

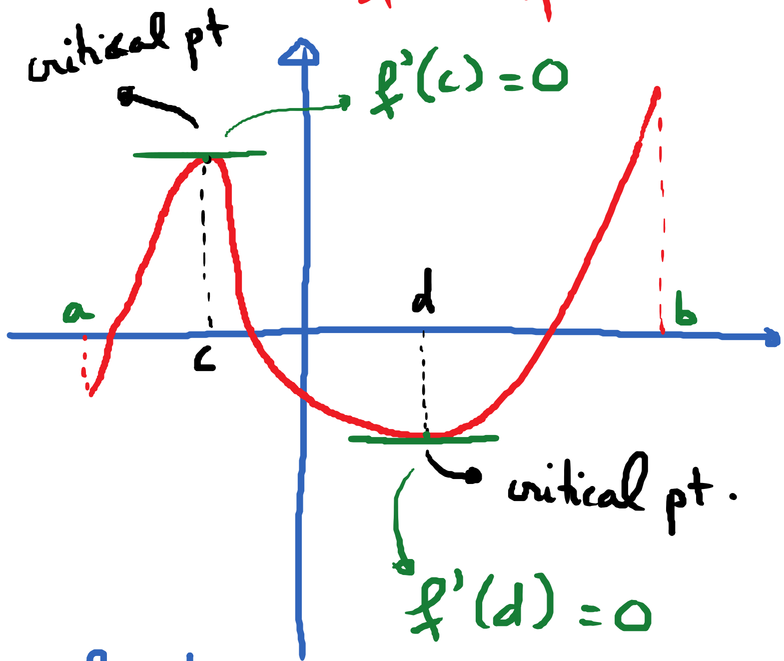
E.g. Given $f(x) = x^3 - 6x^2 + 9x + 1$.

Consider f over the closed interval: $[0,5]$.

Q: How do we find the absolute max and abs. min of f over $[0,5]$.

Critical Points (Critical numbers) of a function.





Def of a critical point of a function:

Let c be a number in the domain of the function f.
We say that c is a critical number of f if one of the followings is true

(1) $f'(c) = 0$.

Or (2) $f'(c)$ is undefined.

To find critical #'s of a function, all we need to do is to take the derivative and find the values of x within the domain of the function at which f' is zero or undefined.

E.g. $f(x) = x^3 - 6x^2 + 9x + 1$.

Find all critical points of f.

$f'(x) = 3x^2 - 12x + 9$.

critical points: $f' = 0$; f' undefined ← never happen

$f' = 0$: $3x^2 - 12x + 9 = 0$

$3(x^2 - 4x + 3) = 0$

$3(x-1)(x-3) = 0$

$\boxed{x = 1}$; $\boxed{x = 3}$

Conclusion: Critical #'s of f are : 1, 3.

Ex. Find the critical points of the given function.

(a) $f(x) = \dfrac{4x}{1+x^2}$

(b) $g(x) = 4\sqrt{x} - x^2$

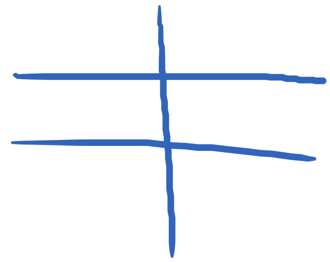
(c) $h(x) = x^{3/5} \cdot (4-x)$.

(d) Could you find the formula for a function that has no critical points?

(d) $\boxed{y = \ln x}$. Domain: $\boxed{(0, \infty)}$

$y' = \frac{1}{x}$ undefined at $x = 0$

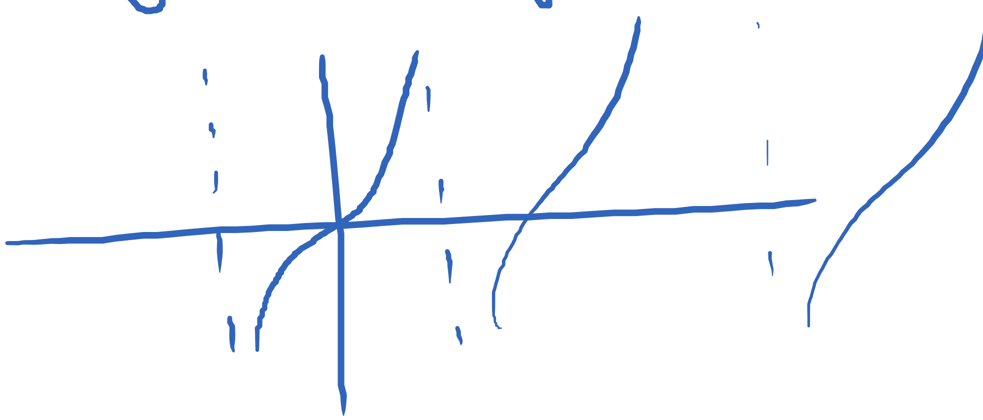
$y = \frac{1}{\sqrt{x}}$ $(0, \infty)$ → $x^{-1/2}$



$y' = -\frac{1}{2} x^{-3/2} = -\frac{1}{2\sqrt{x^3}}$

$\boxed{y = e^x}$, $y' = e^x$

$y = \tan x$; $y' = \sec^2 x = \frac{1}{\cos^2 x}$



$y = \tan^{-1} x$; $y' = \frac{1}{1 + x^2}$

(b) $g(x) = 4\sqrt{x} - x^2$. Domain: $[0, \infty)$.

$$g'(x) = \frac{2}{\sqrt{x}} - 2x$$

$g' = 0$

g' undefined

g' is undefined when $x = 0$.

And $x = 0$ belongs to the domain of g.

So, $x = 0$ is a critical point of g.

Now consider $g' = 0$.

$$\frac{2}{\sqrt{x}} - 2x = 0$$

$$\rightarrow \frac{2}{\sqrt{x}} = 2x \rightarrow \frac{1}{\sqrt{x}} = x \rightarrow x\sqrt{x} = 1$$

$$\rightarrow x^{3/2} = 1 \rightarrow x = 1 \text{ (in domain)}$$

So, $x = 1$ is also a critical point of g.

<u>Conclusion:</u> Critical points are $x = 0$; $x = 1$.

Solved Rest in Class.

Closed Interval Method: to find the absolute max and absolute min of a function f on a closed interval $[a,b]$.

1. Find all the critical points of f within $[a,b]$.
2. Evaluate f at the critical points in 1
3. Evaluate f at the endpoints $x=a$ and $x=b$.
4. The largest value in 2 and 3 will be the absolute max value of f in $[a,b]$
   The smallest value in 2 and 3 will be the absolute min value of f in $[a,b]$

$\boxed{\text{Abs max } (5,21);\ \text{Abs min } (0,1);(3,1)}$

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Ex: (1) $f(x) = x^3 - 6x^2 + 9x + 1$ on $[0,5]$

Find abs max/min of f on $[0,5]$

(2) $g(x) = x \cdot e^{-x^2/8}$ on $[-1,4]$.

Find abs max/min of g on $[-1,4]$

Solved this in class!

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(1) Medical field / Cancer / Surgery.

(2) Physics /

(3) Software / Cryptography.

(4) Civil Engineer.

(5) Astronomy.

(6) AI.