4.4. Rolle's Theorem and the Mean Value Theorem.

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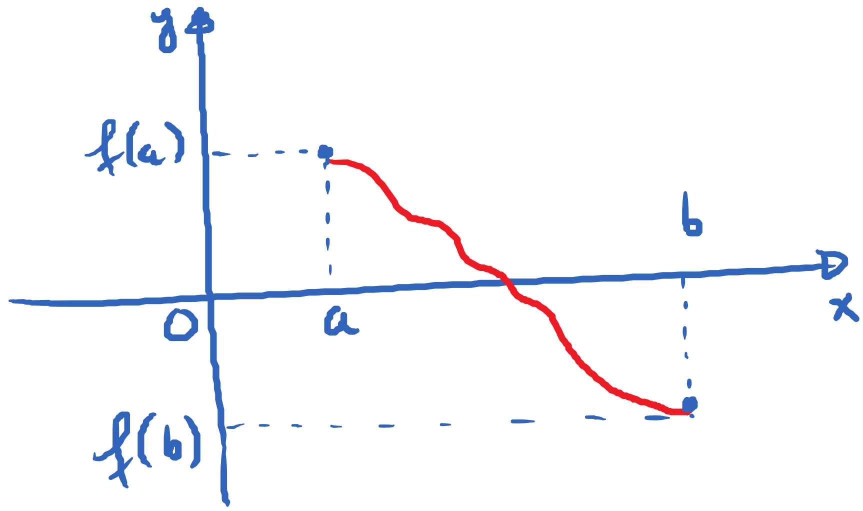
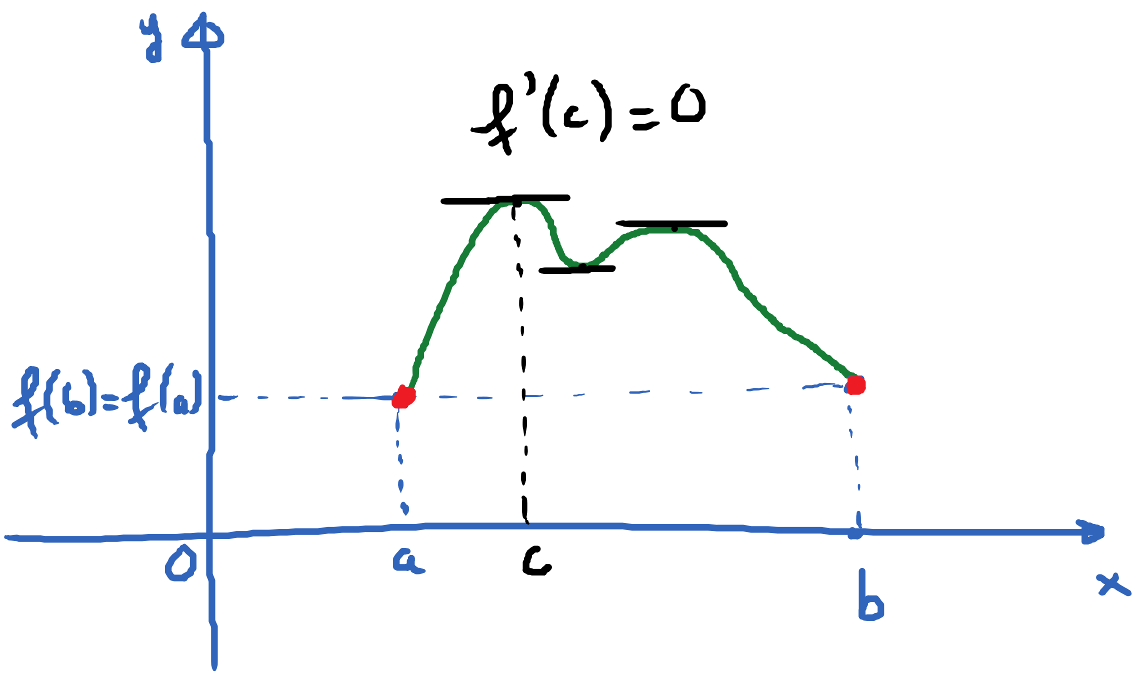
Ex.





Rolle's Theorem:

f: function on the interval $[a,b]$.

① f is continuous on $[a,b]$

② f is differentiable on (a,b)

③ $\boxed{f(a) = f(b)}$

Hypothesis of the Theorem

<u>Conclusion:</u> There exists a number c in (a,b) such that $f'(c) = 0$.

Ex. $f(x) = x^3 - 4x$; on $[-2,2]$

ⓐ Verify that f satisfies all the hypotheses of Rolle's Theorem.

ⓑ Find the value(s) of c such that $f'(c) = 0$.

ⓑ $f'(x) = 3x^2 - 4 = 0 \longrightarrow x = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}}$

$x = \pm\frac{2\sqrt{3}}{3}$

(a) (1) f is continuous on $[-2,2]$ b/c it is a polynomial

(2) f is diff. on $(-2,2)$. Derivative exists everywhere

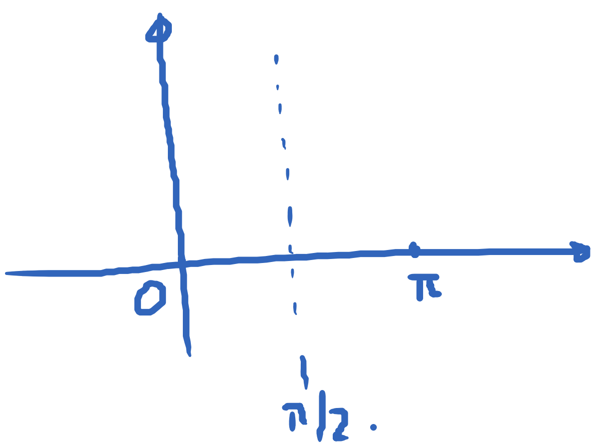
(3) $f(-2)=0$

$f(2)=0$.

$\longrightarrow$ It satisfies Rolle's Theorem.

Ex. $f(x)=\tan(x)$; $f'(x)=\sec^2 x$

$f(0)=0$; $f(\pi)=0$



There is no number c in $(0,\pi)$ such that $f'(c)=0$

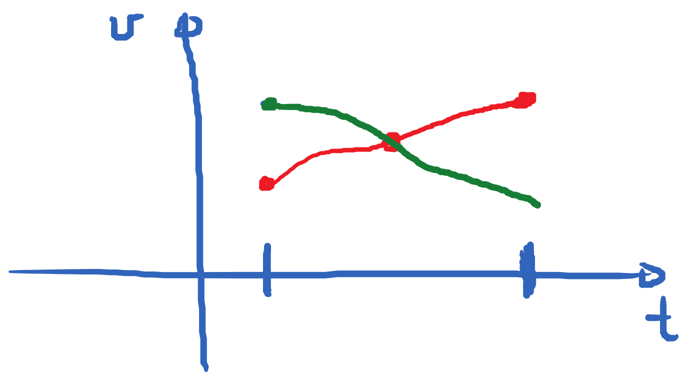
We can't apply Rolle's Thm here b/c f is discont. at $x=\frac{\pi}{2}$.

E.g. 2 runners start a race at the same time. They finish in a tie.

Q: T or F $\boxed{\text{at some point during the race, they have the same speed.}}$

$v_1(t)$: speed of 1^{st} runner

$v_2(t)$: ______ 2^{nd} runner.



Rolle's Theorem:

there is c in (a,b):

$f'(c) = 0$

$s_1'(c) - s_2'(c) = 0$

$s_1'(c) = s_2'(c)$

$s_1(t)$: position of 1^{st} runner.

$s_2(t)$: position of 2^{nd} runner.

$s_1(a) = s_2(a)$; $s_1(b) = s_2(b)$

$f(t) = s_1(t) - s_2(t)$. $f(a) = 0$; $f(b) = 0$

$f(a) = f(b)$

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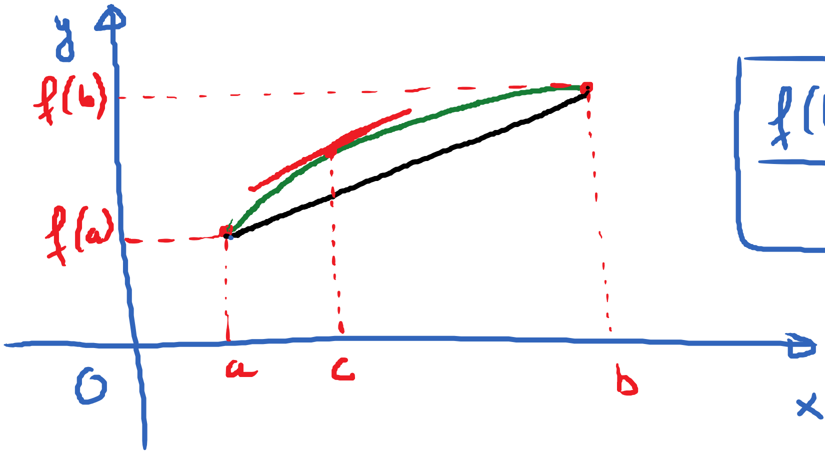
f — cont. on $[a,b]$; diff. on (a,b)

$$\boxed{\frac{f(b)-f(a)}{b-a}} = f'(c)$$



MVT:

f : function on $[a,b]$
f : continuous on $[a,b]$
f : differentiable on (a,b)

} Hypothesis of MVT

Conclusion: there exists a point c in (a,b) s.t.

$$\boxed{f'(c) = \frac{f(b)-f(a)}{b-a}}$$

Ex. 1 $f(x) = \ln x$; on $[1, 4]$

(a) Verify that all the hypotheses of MVT are satisfied.

(b) Find the value(s) of c that satisfy the conclusion of MVT. $c = \frac{\ln(4)}{3}$

2 Suppose that $f(0) = -3$. f is diff., cont. everywhere. $f'(x) \leq 5$ for all values of x. Q: How large can $f(2)$ possibly be.

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1 (a) ✓

(b) $$\frac{f(b) - f(a)}{b - a} = f'(c)$$

$$\frac{\ln(4) - \ln(1)}{4 - 1} = \frac{1}{c}$$

$$\frac{\ln 4}{3} = \frac{1}{c} \rightarrow \boxed{c = \frac{3}{\ln(4)}}$$

$$\frac{f(2) - f(0)}{2 - 0} = f'(c)$$

$$\frac{f(2) + 3}{2} = f'(c) \leq 5$$

$$\frac{f(2) + 3}{2} \leq 5$$

$$f(2) + 3 \leq 10$$

$$f(2) \leq 7$$