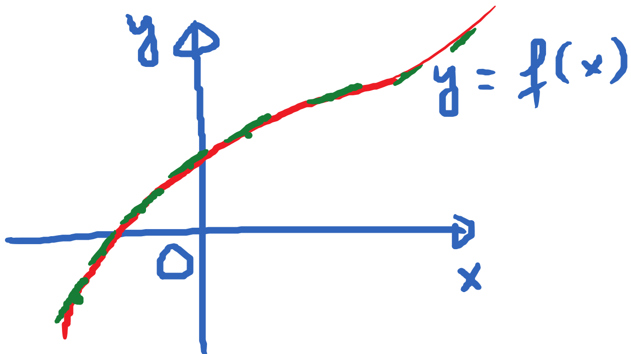
4.5. Derivatives and the shape of a graph.

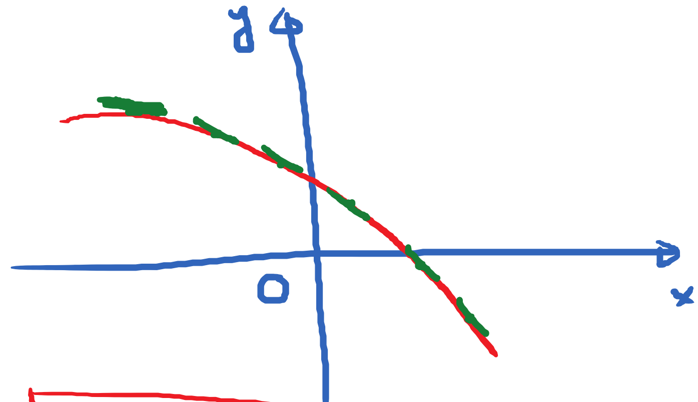
Monday, March 19, 2018 8:20 AM

How the derivative tells us about the shape of a graph

* What does f' tell us about f?

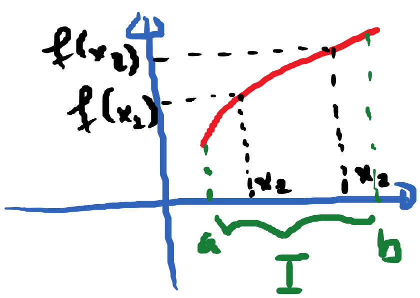
Increasing / Decreasing





$$\boxed{f' > 0 \longleftrightarrow \underbrace{f\uparrow}_{\text{increasing}}} \qquad \boxed{f' < 0 \longleftrightarrow \underbrace{f\downarrow}_{\text{decreasing}}}$$

<u>Proof</u>: If $f' > 0$ on an interval I, then f is increasing on I.



Choose 2 random points x_1 and x_2 in I. $x_1 < x_2$.

<u>WTS</u>: $f(x_2) > f(x_1)$

By MVT:

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = f'(c) \text{ for some } c \text{ in } (x_1, x_2)$$

But $f'(c) > 0$.

So, $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} > 0.$

So, $f(x_2) - f(x_1) > 0$

So, $f(x_2) > f(x_1)$

E.g. $f(x) = 3x^4 - 4x^3 - 12x^2 + 5.$

Find the intervals on which f is increasing | decreasing

Step 1: find $f'(x)$. $f'(x) = 12x^3 - 12x^2 - 24x.$

Find critical points $\left\langle \begin{array}{l} f' \text{ is undefined (does not happen)} \\ f' = 0 \end{array} \right.$

Monday, March 19, 2018 8:41 AM

$f' = 0$. $\underbrace{12x^3 - 12x^2 - 24x} = 0$

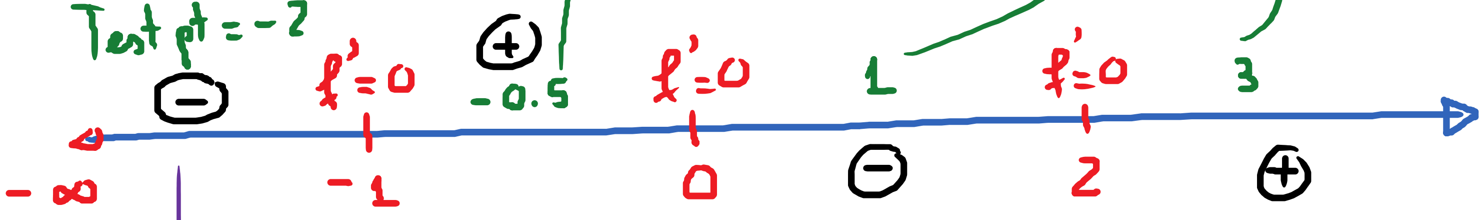
$12x(x^2 - x - 2) = 0$

plug in f'

$12x(x-2)(x+1) = 0$

$x = 0$; $x = 2$; $x = -1$

Test pt $= -2$



f decreasing

f increasing

f decreasing

f increasing

local max @ 0

value of local max : $f(0)$

local min @ -1

value of local min $f(-1)$

local min @ 2

value of local min $f(2)$

Answer: Decreasing on : $(-\infty, -1) \cup (0, 2)$

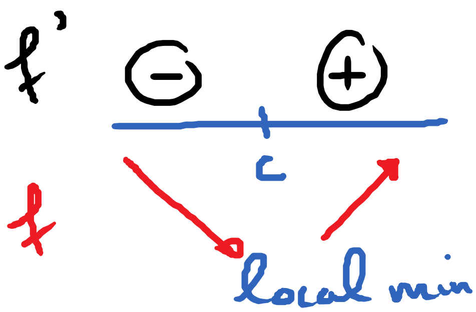
Increasing on : $(-1, 0) \cup (2, \infty)$

⟶ First Derivative Test.

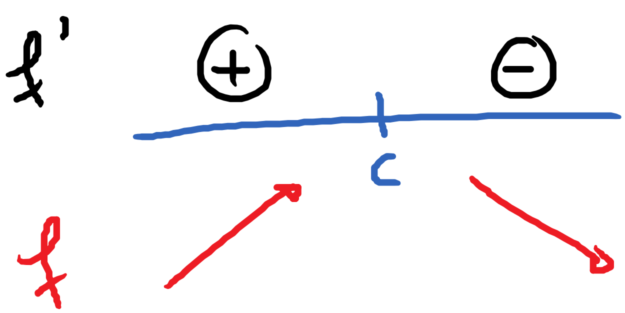
The first derivative test.

If c is a critical point of a continuous function, then:

① If the sign of f' changes from negative to positive at c, then f has a local min at c.

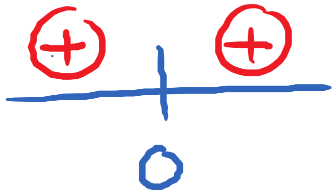


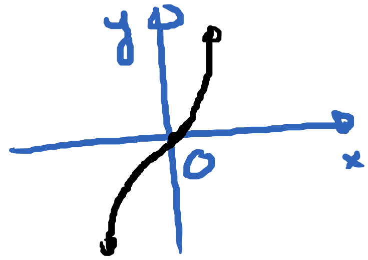
② If f' changes from ⊕ to ⊖ at c, then f has a local max at c.



Monday, March 19, 2018 8:55 AM

$f(x) = x^3$; $f'(x) = 3x^2 = 0$ when $x = 0$





③ If f' does not change sign at c, f has neither a local min nor a local max at c.

Ex: $g(x) = x + 2\sin x$ on $\boxed{(0, 2\pi)}$ → open interval.

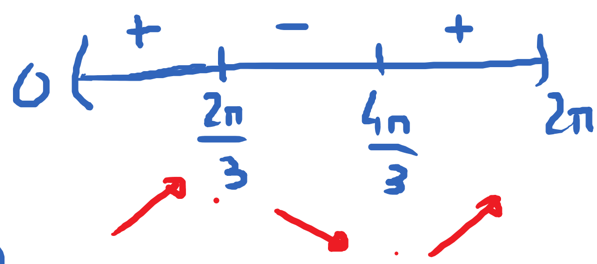
(x-coord, y-coord)

Q: Find the local min and the local max of g on this interval.

(x-coord, y-coord)

Local max $\left(\frac{2\pi}{3}, \frac{2\pi}{3} + \sqrt{3}\right)$

Local min $\left(\frac{4\pi}{3}, \frac{4\pi}{3} - \sqrt{3}\right)$



$g'(x) = 1 + 2\cos x = 0$

$\boxed{\cos x = -\frac{1}{2}}$ on $(0, 2\pi)$

$x = \frac{2\pi}{3}$, $x = \frac{4\pi}{3}$.