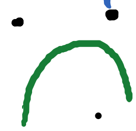
What does f'' tell us about f?

Concavity

positive

concave up



negative

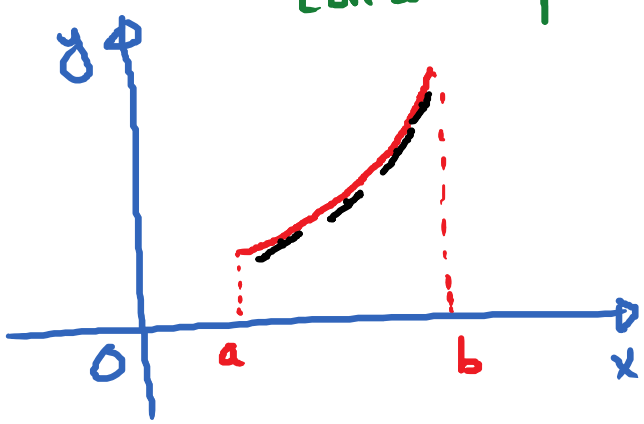
concave down

ⓐ If $\boxed{f''(x) > 0}$ for every x in an interval I, then f is $\boxed{\text{concave up}}$ on I.

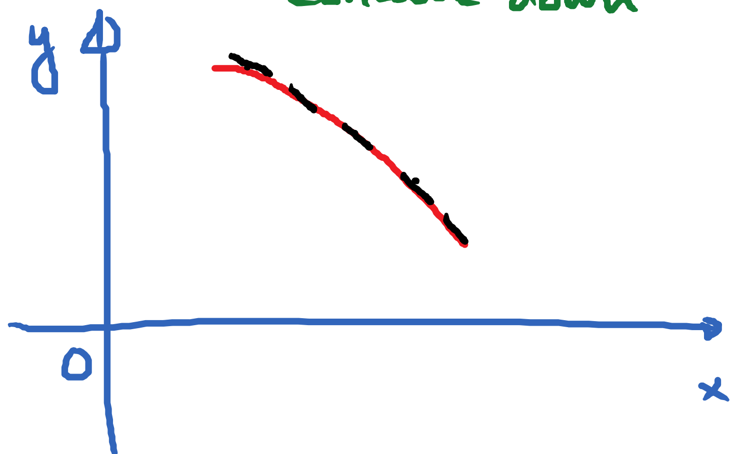
ⓑ If $\boxed{f''(x) < 0}$ for every x in an interval I, then f is $\boxed{\text{concave down}}$ on I

Why?

Concave up



Concave down



Slopes are increasing, $f' \uparrow$

$\longleftrightarrow f'' > 0$

Slopes are decreasing, $f' \downarrow$

$\longleftrightarrow f'' < 0$

E.g. $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$.

Find intervals on which f is concave up | down.

$$f'(x) = 12x^3 - 12x^2 - 24x$$

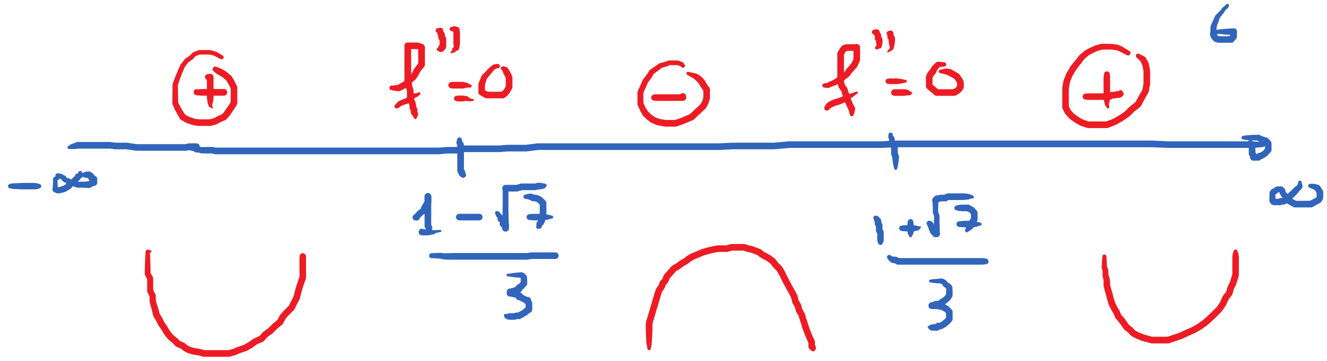
$$f''(x) = 36x^2 - 24x - 24 = 0$$

$$3x^2 - 2x - 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{4 + 24}}{6} = \frac{2 \pm \sqrt{28}}{6}$$

$$= \frac{2 \pm 2\sqrt{7}}{6} = \frac{1 \pm \sqrt{7}}{3}$$



Concave up on: $\left(-\infty, \frac{1-\sqrt{7}}{3}\right) \cup \left(\frac{1+\sqrt{7}}{3}, \infty\right)$

Concave down on: $\left(\frac{1-\sqrt{7}}{3}, \frac{1+\sqrt{7}}{3}\right)$

Def: A point $P(c, f(c))$ is called an inflection point of $y = f(x)$ if f changes concavity when it passes through P.

$f'' < 0$ $\quad f'' = 0$ $\quad f'' > 0$

down $\quad c \quad$ up

on

$f'' > 0$ $\quad f'' = 0$ $\quad f'' < 0$

up $\quad c \quad$ down

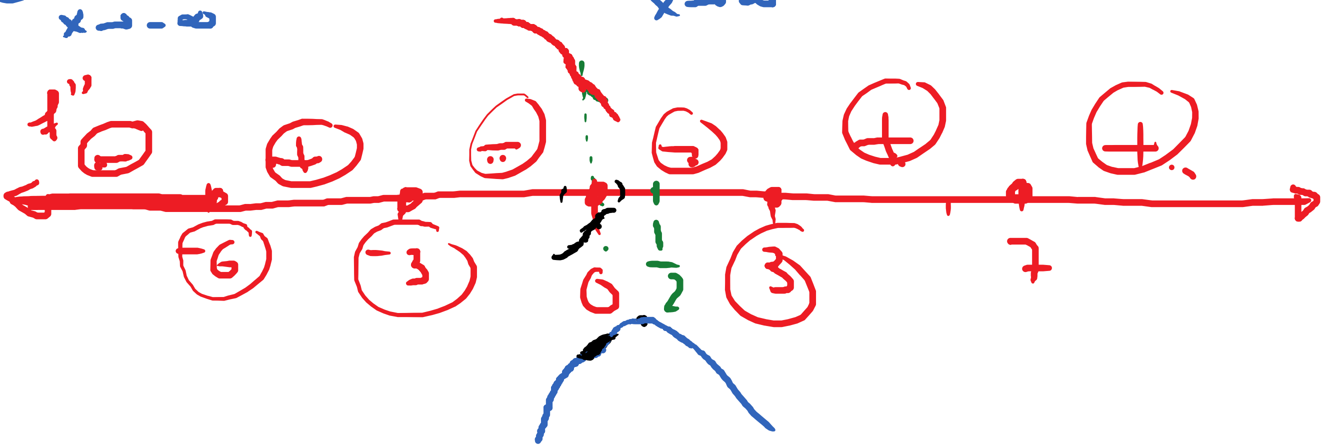
E.x. Sketch a graph of a function that satisfies all the followings

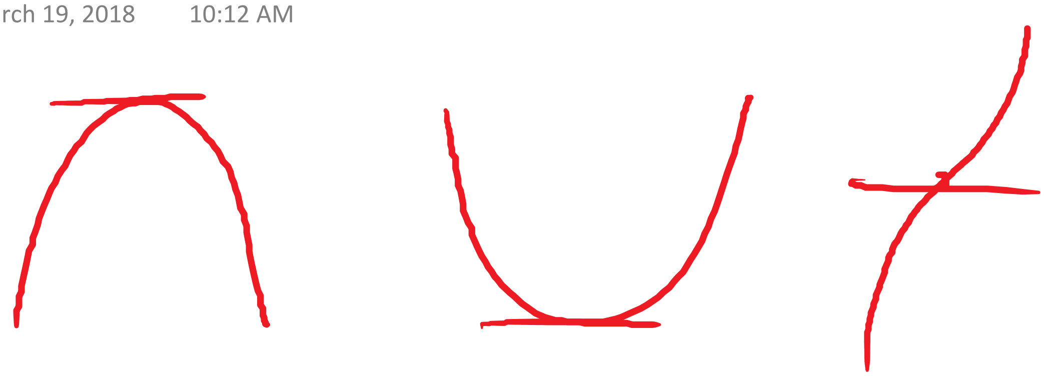
① $f'(x) > 0$ on $(-\infty, 1)$; $f'(x) < 0$ on $(1, \infty)$

② $f''(x) > 0$ on $(-\infty, -2)$ and $(2, \infty)$

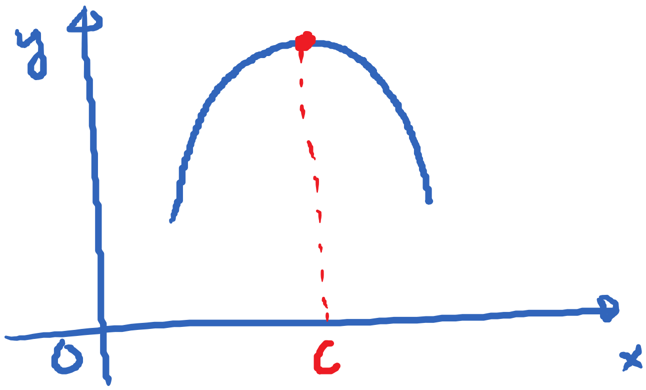
$f''(x) < 0$ on $(-2, 2)$

③ $\lim\limits_{x \to -\infty} f(x) = -2$; $\lim\limits_{x \to \infty} f(x) = 0$

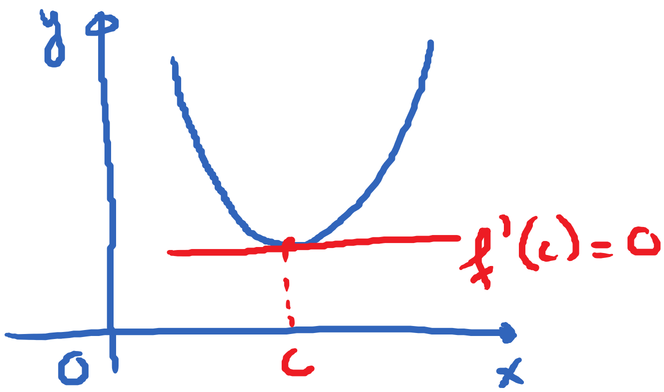




Second Derivative Test:



If $f'(c) = 0$ and $f''(c) < 0$, then c corresponds to a local max of f.



If $f'(c) = 0$ and $f''(c) > 0$, then c corresponds to a local min of f.

Note: If $f'(c) = f''(c) = 0$, we cannot make any conclusion about local max/min at c.