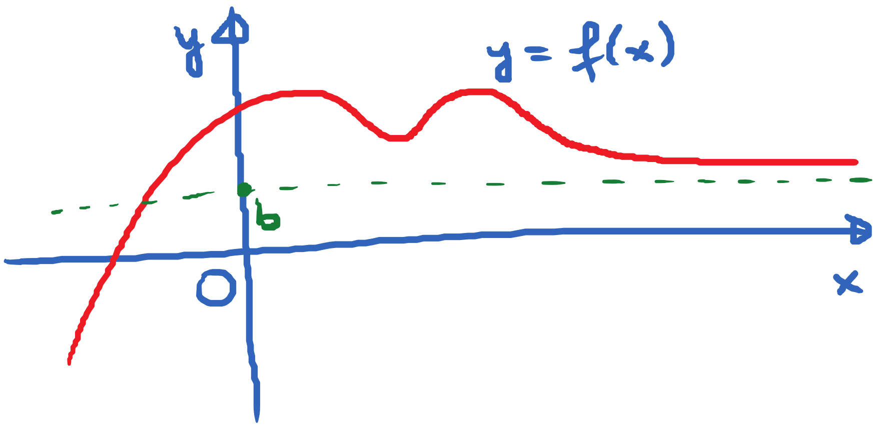
4.6. Limits at Infinity and Asymptotes.

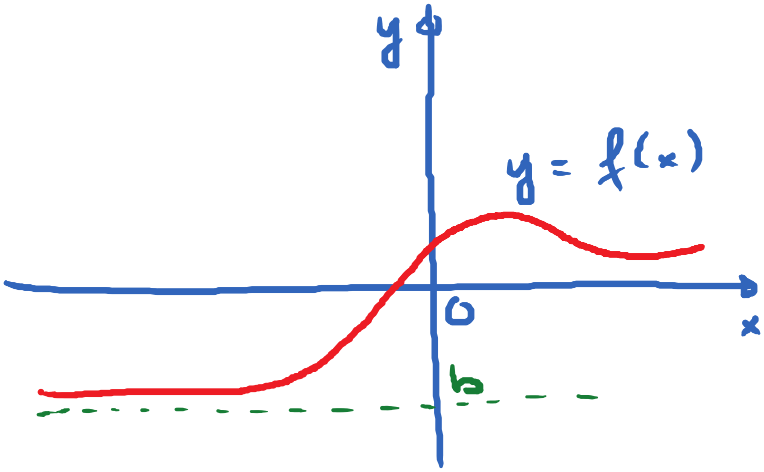
Wednesday, March 21, 2018 8:12 AM

Goals: Find limits at infinity and find H.A. and V.A. of functions.



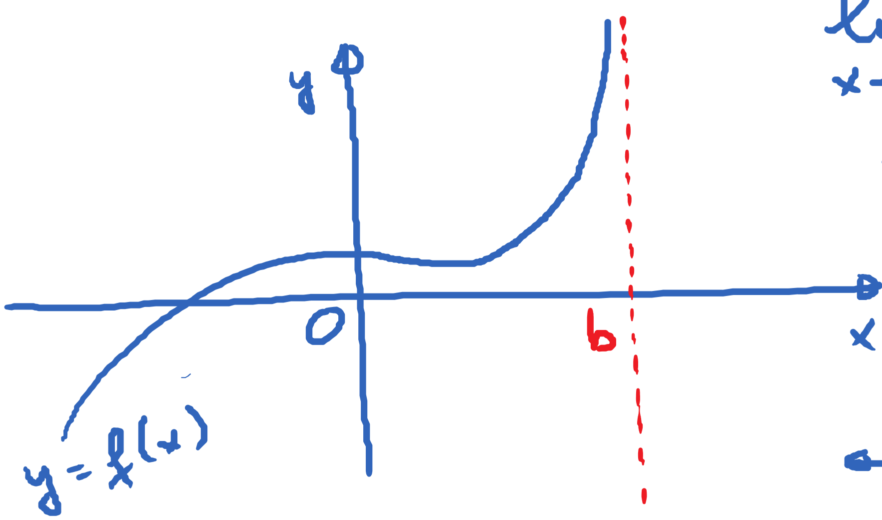
$y = b$ is a H.A. of $y = f(x)$

$$\lim_{x \to \infty} f(x) = b$$



$$\lim_{x \to -\infty} f(x) = b$$

$y = b$ is a H.A. of f.



$$\lim_{x \to b^-} f(x) = \infty$$

$x = b$ is a V.A. of f

$$\lim_{x \to b^\pm} f(x) = \pm\infty \iff x = b \text{ is a V.A. of } f.$$

Basic Limits at ∞

(1) $f(x) = x^n$; n is a positive integer

$$\lim_{x \to \infty} [x^n] = \infty$$

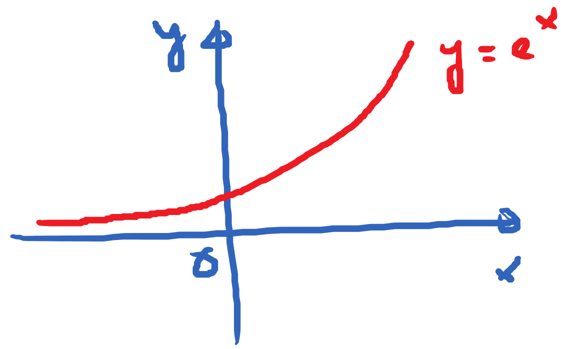
$$\boxed{\lim_{x \to -\infty} [x^n]} = \begin{cases} \infty & \text{if } n \text{ is even} \\ -\infty & \text{if } n \text{ is odd.} \end{cases}$$

(2) $f(x) = \dfrac{1}{x^n}$; n : positive integer

$$\boxed{\lim_{x \to \infty} \left[\frac{1}{x^n}\right]} = 0 \quad ; \quad \lim_{x \to -\infty} \frac{1}{x^n} = 0$$

$$\boxed{\lim_{x \to \pm\infty} \left[\frac{1}{x^n}\right] = 0}$$

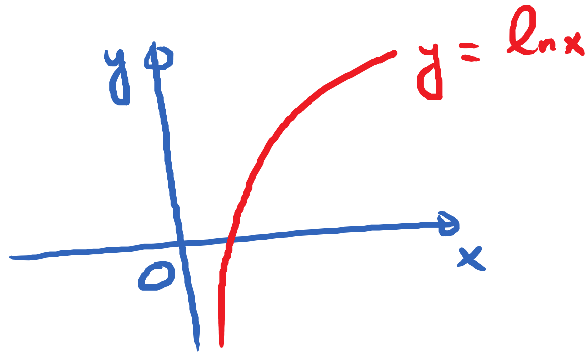
③ $f(x) = e^x$



$$\lim_{x \to \infty} \left[e^x \right] = \infty$$

$$\lim_{x \to -\infty} \left[e^x \right] = 0$$

④ $f(x) = \ln x$

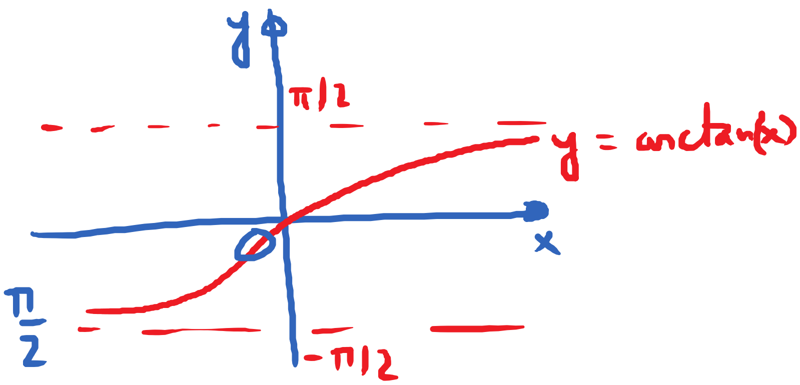


$$\lim_{x \to \infty} \left[\ln x \right] = \infty$$

$$\lim_{x \to 0} \left[\ln x \right] = -\infty$$

⑤ $f(x) = \arctan(x)$

$$\lim_{x \to \infty} \left[\arctan(x) \right] = \frac{\pi}{2}$$



$$\lim_{x \to -\infty} \left[\arctan(x) \right] = -\frac{\pi}{2}$$

E.g. $\lim_{x \to \infty} \frac{x^2 - 2x + 5}{3x^2 + 7} \approx \frac{x^2}{3x^2} = \frac{1}{3}$

$$\lim_{x \to \infty} \frac{(x^2 - 2x + 5)/x^2}{(3x^2 + 7)/x^2}$$

$$= \lim_{x \to \infty} \frac{\frac{x^2 - 2x + 5}{x^2}}{\frac{3x^2 + 7}{x^2}} = \lim_{x \to \infty} \frac{1 - \frac{2}{x} + \frac{5}{x^2}}{3 + \frac{7}{x^2}}$$

0 0 0

$$= \boxed{\frac{1}{3}}$$

$$\lim_{x \to \infty} \frac{x^2 + 10}{3x^3 + x^2 + 7} \approx \frac{x^2}{3x^3} = \frac{1}{3x} = 0$$

$$\lim_{x\to\infty} \frac{x^3+1}{x^2+2018} = \infty$$

$$\lim_{x\to-\infty} \frac{x^3+1}{x^2+2018} \approx \frac{x^3}{x^2} = x = -\infty$$

$$\lim_{x\to-\infty} \frac{x^{2018}+x^3+10}{x^{2000}+x^{1999}-5} \approx \frac{x^{2018}}{x^{2000}} = x^{18}$$

$$= \infty$$

$$\lim_{x\to-\infty} \frac{x^{2017}+x^{2016}+5}{x^{2011}+x^5+7} = \infty$$

<u>E.g.</u>

$$\lim_{x\to\infty} \frac{3x+2}{\sqrt{4x^2+5}} = \lim_{x\to\infty} \frac{3x+2}{(4x^2+5)^{1/2}}$$

$$\approx \frac{3x}{(4x^2)^{1/2}} = \frac{3x}{\sqrt{4x^2}} = \frac{3\cancel{x}}{2\cancel{x}}$$

$$= \frac{3}{2}$$

$$\lim_{x \to -\infty} \frac{3x+2}{\sqrt{4x^2+5}} = -\frac{3}{2}$$

E.g. $\lim_{x \to \infty} \frac{1-e^x}{1+2e^x} \approx \frac{-\cancel{e^x}}{2\cancel{e^x}} = -\frac{1}{2}$.

$$\lim_{x \to \infty} \frac{1-e^x}{1+2e^{2x}} \approx -\frac{1}{2e^x} \longrightarrow ?$$

0

when $x \to \infty$

$$\boxed{\frac{0}{0}} \quad \lim_{x \to a} \frac{f(x)}{g(x)} \approx \frac{0}{0}$$

$$\lim_{x \to 0} \frac{x}{e^x - 1} = \infty$$

$x \to 0$, $e^x - 1 \to 0$

$$\lim_{x \to 0} \frac{x^2}{10x^2} \; \frac{0}{0} \to \frac{1}{10}$$

$$\lim_{x \to 0} \frac{2018x^2}{x^2} = 2018$$

$\frac{0}{0}$

E.g. $\lim_{x\to\infty} \arctan(e^x) = \frac{\pi}{2}$.

$\frac{4}{0} = \infty$

$f(x) = \frac{x^2-9}{x^2-4x+3} \to \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}\boxed{(x-1)}} = \frac{6}{2} = 3$

H.A: $y = 1$ b/c $\lim_{x\to\infty} \frac{x^2-9}{x^2-4x+3} = 1$

V.A. $x = 1$

Hole: @ $x = 3$