5.1 and 5.2

Monday, April 9, 2018 8:04 AM

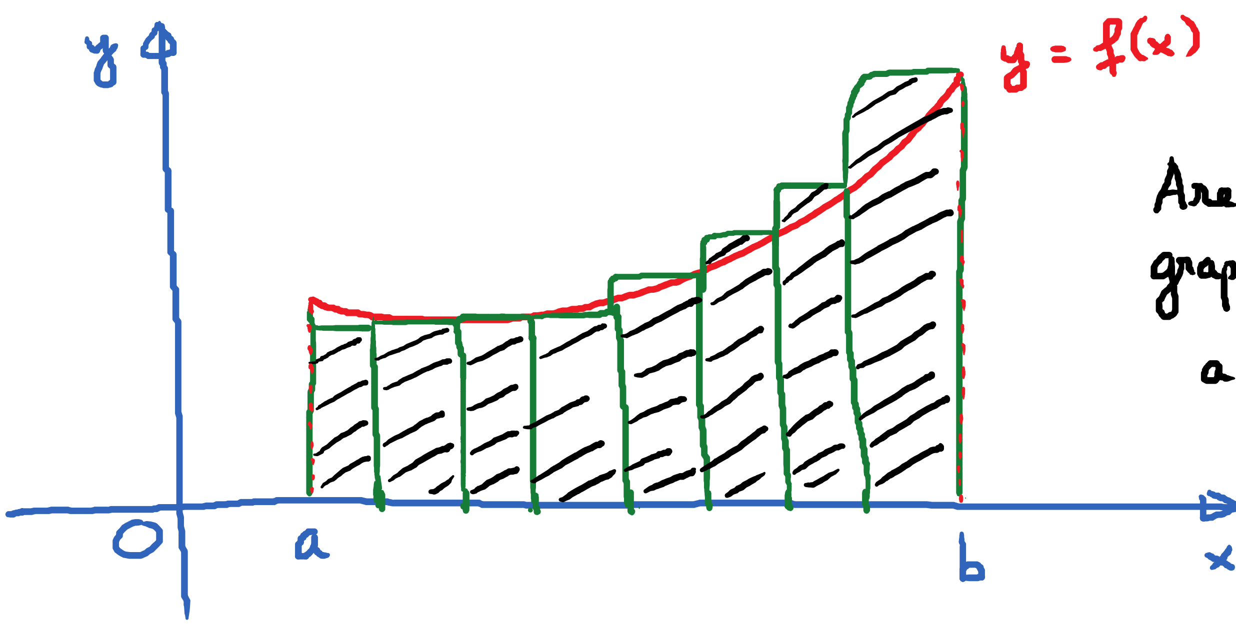
Recall: Antiderivatives/ Indefinite Integrals.

$\int \boxed{f(x)}\,dx$ = the most general antiderivative of $f(x)$

$= F(x) + C$

where $F'(x) = f(x)$

$\longrightarrow$ look for a function $F(x)$ whose derivative is equal to $f(x)$.



Area under the graph of $y = f(x)$; $a \leq x \leq b$

Monday, April 9, 2018 8:15 AM

Summation Notation

Summation Notation (Sigma Notation)

$$1+2+3+4+5+\cdots+1000=\sum_{i=1}^{1000} \boxed{i}$$

$$\sum_{i=1}^{999} i^2 = (1)^2+(2)^2+\cdots+(999)^2$$

$$\sum_{i=3}^{101} \frac{1}{i(i+1)} = \frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\cdots+\frac{1}{101\cdot 102}.$$

Summation Formula.

<u>Gauss.</u> $1+2+3+\cdots+100 \doteq ?$

$$\begin{array}{rl} & S = 1+2+3+\cdots+100 \\ + & S = 100+99+98+\cdots+1 \\ \hline & 2S = \underbrace{101+101+101+\cdots+101}_{\text{100 terms of 101}} \end{array}$$

$$2S = 100\cdot 101 \longrightarrow S = \frac{100\cdot 101}{2} = 50\cdot 101$$

$$\boxed{S = 5050}$$

Monday, April 9, 2018 8:27 AM

$$S = \underbrace{1 + 2 + \cdots + n}_{\sum_{i=1}^{n} i} = \frac{n(n+1)}{2}$$

$1^{\underline{st}}$ Summation formula : $\boxed{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}}$ I

II c : constant . $\boxed{\sum_{i=1}^{n} c = nc}$

$$\underbrace{\underset{i=1}{c} + \underset{i=2}{c} + \underset{i=3}{c} + \cdots + \underset{i=n}{c}}_{\text{add } c \text{ together } n \text{ times}} = nc$$

III $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

Monday, April 9, 2018 8:32 AM

$$\sum_{i=1}^{30} (i+2)^2 = \sum_{i=1}^{30} (i^2 + 4i + 4)$$

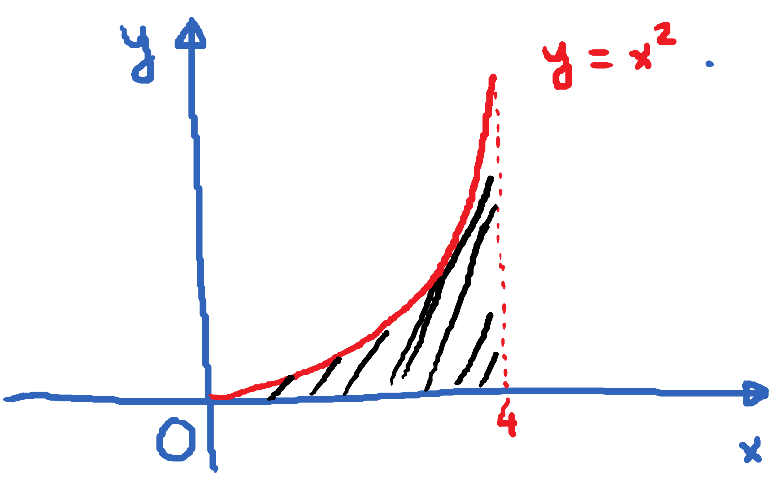
$$= \sum_{i=1}^{30} i^2 + \sum_{i=1}^{30} 4i + \sum_{i=1}^{30} 4$$

$$= \sum_{i=1}^{30} i^2 + 4 \sum_{i=1}^{30} i + \sum_{i=1}^{30} 4$$

$$= \frac{30 \cdot 31 \cdot 61}{6} + 4 \cdot \frac{30 \cdot 31}{2} + 4 \cdot 30$$

$$= 11435$$

Using Sums to Approximate Areas.

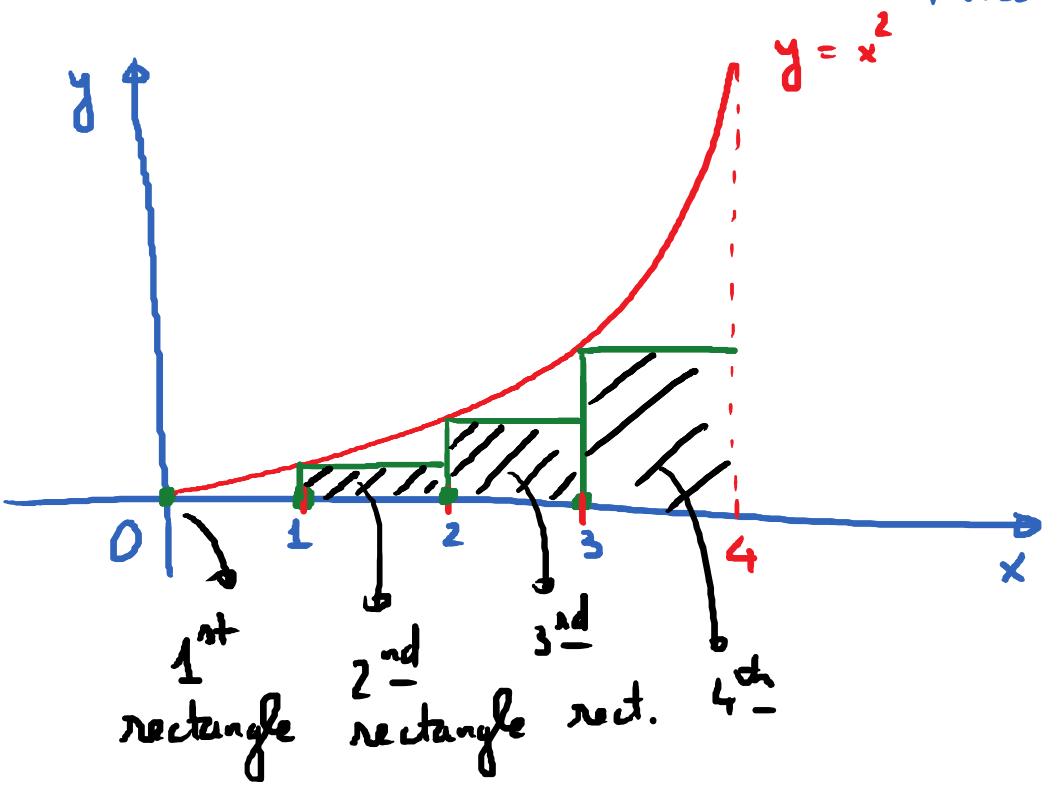


Consider $f(x) = x^2$ over the interval $[0, 4]$

Q: Approximate the area under the graph using L_4 and R_4.

Monday, April 9, 2018 8:59 AM

L_4: left endpoint approximation (left Riemann Sum with 4 subintervals)



L_4 = Sum of the areas of these 4 rectangles

width of each rectangle $= \frac{4}{4} = 1$

$$= f(0)\cdot 1 + f(1)\cdot 1 + f(2)\cdot 1 + f(3)\cdot 1$$

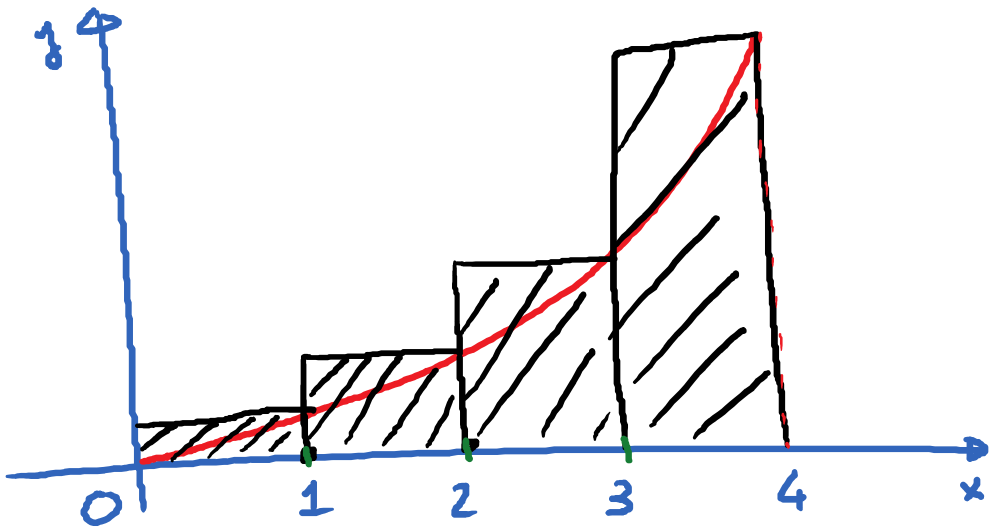
$$= \sum_{i=1}^{4} f(i-1)\cdot 1 = 0\cdot 1 + 1\cdot 1 + 4\cdot 1 + 9\cdot 1$$

$$= \boxed{14}$$

$L_4 = 14 \longrightarrow 14 <$ Area.

Monday, April 9, 2018 9:04 AM

Find R_4. (Right endpoint approximation or Right Riemann Sum)



$R_4 = f(1)\cdot 1 + f(2)\cdot 1 + f(3)\cdot 1 + f(4)\cdot 1$

$= \sum_{i=1}^{4} f(i)\cdot 1 = 30.$

$\longrightarrow L_4 < \text{Area} < R_4$

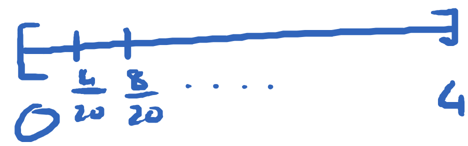
$\longrightarrow 14 < \text{Area} < 30$

* Consider $n = 20$. L_{20} ; R_{20}.

$L_{20} \rightarrow$ width of each rectangle : $\dfrac{4}{20} = \dfrac{1}{4}$.

$$L_{20} = f\left(\frac{0}{20}\right)\cdot\frac{1}{4} + f\left(\frac{1}{20}\right)\cdot\frac{1}{4} + \cdots + f\left(\frac{19}{20}\right)\cdot\frac{1}{4}$$

$$= \sum_{i=1}^{20} f(x_{i-1})\cdot\frac{1}{4}$$



$L_{20} = 19.76$

$$R_{20} = f\left(\frac{1}{20}\right)\cdot\frac{1}{4} + \cdots + f\left(\frac{20}{20}\right)\cdot\frac{1}{4}.$$

$$= \sum_{i=1}^{20} f(x_i)\cdot\frac{1}{4}.$$

$R_{20} = 22.96$

$L_{20} <$ Area $< R_{20}$

$19.76 <$ Area < 22.96

Monday, April 9, 2018 9:15 AM

$n = 50$. width of rectangle $= \frac{4}{50}$.

$$L_{50} = \sum_{i=1}^{50} f(x_{i-1}) \cdot \frac{4}{50} = 20.6976$$

$$R_{50} = \sum_{i=1}^{50} f(x_i) \cdot \frac{4}{50} = 21.9776$$

$L_{50} < \text{Area} < R_{10}$. $\longrightarrow$ first digit of area $= 2$

$n = 100 \longrightarrow$ width $= \frac{4}{100}$.

$$L_{100} = \sum_{i=1}^{100} f(x_{i-1}) \cdot \frac{4}{100} = 21.0144$$

$$R_{100} = \sum_{i=1}^{100} f(x_i) \cdot \frac{4}{100} = 21.6544$$

$$L_{100} < \text{Area} < R_{100}$$

$\longrightarrow$ Area ≈ 21 (something)