Monday, April 16, 2018 9:24 AM

7) $\int x(2x+5)^8\,dx$ (Use u - sub)

8) $\int \dfrac{dx}{(1+\sqrt{x})^4}$

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1) $\int x^2 \cdot e^{\boxed{x^3}}\,\boxed{dx}$

Let $u = x^3$

$du = 3x^2\,dx$

$dx = \dfrac{du}{3x^2}$

$$\int \cancel{x^2} \cdot e^u \,\frac{du}{3\cancel{x^2}}$$

$$\int \frac{e^u}{3}\,du = \frac{1}{3}\int e^u\,du = \frac{1}{3}\cdot e^u + C$$

$$= \frac{1}{3}e^{x^3} + C.$$

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(2) $\int \boxed{e^x} \cos(\boxed{e^x}) \boxed{dx}$

let $u = e^x$

$du = e^x\, dx$

$= \int \cos(\overset{u}{e^x}) \underbrace{e^x\, dx}_{du}$

$= \int \cos(u)\, du = \sin(u) + C = \sin(e^x) + C$

(3) $\int \frac{\sin(t)}{\cos^3(t)} dt$

let $u = \cos(t)$

$du = -\sin(t)\, dt$

$dt = \frac{du}{-\sin(t)}$

$\int \frac{\cancel{\sin(t)}}{u^3} \cdot \frac{du}{\cancel{-\sin(t)}}$

$-\int \frac{du}{u^3} = -\int u^{-3}\, du = -\frac{u^{-2}}{-2} + C$

$= \frac{1}{2u^2} + C$

$= \frac{1}{2\cos^2 t} + C.$

4. $\int y\sqrt{y^2-5}\,dy$

let $u = y^2 - 5$

$du = 2y\,dy$

$dy = \frac{du}{2y}$

$$\int \sqrt{u}\cdot \cancel{y}\cdot \frac{du}{2\cancel{y}}$$

$$\int \frac{\sqrt{u}}{2}\,du = \frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\int u^{1/2}\,du$$

$$= \frac{1}{2}\cdot\frac{2}{3}\cdot u^{3/2} + C = \frac{1}{3}\cdot\left(y^2-5\right)^{3/2} + C.$$

5. $-\int \sqrt{\underbrace{\cot x}_{u}}\cdot \underbrace{\boxed{(-\csc^2 x\,dx)}}_{du}$

$u = \cot x$; $du = -\csc^2 x\,dx$

$$-\int \sqrt{u}\,du = -\int u^{1/2}\,du = -\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \boxed{-\frac{2}{3}\left(\cot x\right)^{3/2} + C}$$

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⑥ $\int \frac{\tan^{-1} x}{1+x^2}\,dx$

Let $u = \tan^{-1} x$

$du = \frac{1}{1+x^2}\,dx$

$$\int u\,du = \frac{u^2}{2} + C = \boxed{\frac{(\tan^{-1} x)^2}{2} + C}$$

⑦ $\frac{1}{2}\int x\,(\boxed{2x+5})^8\,\underbrace{(2\,dx)}_{du}$

Let $\boxed{u = 2x+5}$

$du = 2\,dx$

$$\boxed{x = \frac{u-5}{2}}$$

$$\frac{1}{2}\int x \cdot u^8\,du$$

$$\frac{1}{2}\int \frac{u-5}{2}\cdot u^8\,du = \frac{1}{4}\int (u-5)\cdot u^8\,du$$

$$= \frac{1}{4}\int (u^9 - 5u^8)\,du = \frac{1}{4}\cdot\left(\frac{u^{10}}{10} - 5\cdot\frac{u^9}{9}\right) + C$$

$$= \boxed{\frac{(2x+5)^{10}}{40} - \frac{5(2x+5)^9}{36} + C}$$

(8) $\int \frac{dx}{(\boxed{1+\sqrt{x}})^4}$

Let $\boxed{u = 1 + \sqrt{x}}$ $\rightarrow$ $\boxed{\sqrt{x} = u - 1}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$\rightarrow 2\sqrt{x}\,du = dx$

$$\int \frac{dx}{u^4}$$

$$\int \frac{2\boxed{\sqrt{x}}\,du}{u^4}$$

$$\int \frac{2(u-1)}{u^4}\,du = 2 \cdot \int \frac{u-1}{u^4}\,du$$

$$= 2 \cdot \int (u-1) \cdot u^{-4}\,du = 2 \cdot \int \left(u^{-3} - u^{-4}\right) du$$

$$= 2 \cdot \left(-\frac{1}{2}u^{-2} + \frac{1}{3}u^{-3}\right) + C$$

$$= -\frac{1}{u^2} + \frac{2}{3u^3} + C = -\frac{1}{(1+\sqrt{x})^2} + \frac{2}{3(1+\sqrt{x})^3} + C$$

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u-sub for definite integrals

E.g. $\frac{1}{2}\int_0^4 \sqrt{2x+1}\,\overbrace{(2dx)}^{du}$; $u = 2x+1$

$du = 2dx$

$\frac{1}{2}\int \sqrt{u}\, du$

2 ways to proceed:

1st way: find the new bounds for u.

$x = 0 \longrightarrow u = 2\cdot(0)+1 = 1$

$x = 4 \longrightarrow u = 2\cdot(4)+1 = 9$

$$\frac{1}{2}\int_1^9 \sqrt{u}\, du = \frac{1}{2}\int_1^9 u^{1/2}\, du = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}u^{3/2}}{3}\Bigg|_1^9$$

$$= \frac{(9)^{3/2}}{3} - \frac{(1)^{3/2}}{3}$$

$$= \frac{27}{3} - \frac{1}{3} = 9 - \frac{1}{3} = \boxed{\frac{26}{3}}$$

2nd way to proceed: temporarily ignore bounds for u

$$\frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\int u^{1/2}\,du = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}u^{3/2}}{3}$$

$$= \left.\frac{(2x+1)^{3/2}}{3}\right|_0^4$$

$$= \frac{(9)^{3/2}}{3} - \frac{1}{3} = \frac{26}{3}.$$

Ex:

① $\displaystyle\int_0^1 (3t-1)^{50}\,dt$

③ $\displaystyle\int_0^1 x e^{-x^2}\,dx$

② $\displaystyle\int_1^2 \frac{e^{1/x}}{x^2}\,dx$

④ $\displaystyle\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}}$

Solved in class.