5.6. Integrals that involve Exponential and Log

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Formulas:

I $\boxed{\int e^x\,dx = e^x + C. \quad ; \quad \int e^u\,du = e^u + C}$

If the base is a where $a \neq e$, $a > 0$; $a \neq 1$.

II $\boxed{\int a^x\,dx = \dfrac{a^x}{\ln a} + C \; ; \quad \int a^u\,du = \dfrac{a^u}{\ln a} + C.}$

III $\boxed{\begin{array}{l} \int \frac{1}{x}\,dx = \int \frac{dx}{x} = \ln|x| + C \\ \int \frac{du}{u} = \ln|u| + C \end{array}}$

E.g.

$$\int \frac{dx}{x(\ln x)^2}$$

Let $u = \ln x$

$du = \frac{1}{x}\,dx$

$$\int \frac{du}{u^2} = \int u^{-2}\,du = -\frac{1}{u} + C$$

$$= \boxed{-\frac{1}{\ln x} + C}$$

E.g.

$$\int \frac{dx}{x \ln(x) \ln(\ln(x))}$$

let $u = \ln x$; $du = \frac{1}{x}\,dx$

$$\int \frac{du}{u \ln(u)}$$

let $z = \ln(u)$

$dz = \frac{1}{u}\,du$

$$\int \frac{dz}{z} = \ln|z| + C$$

$$= \ln|\ln(\ln x)| + C$$

2nd way:

$$\int \frac{\overset{du}{dx}}{x\ln(x)\,\boxed{\ln(\ln(x))}_{u}}$$

Let $u = \ln(\ln(x))$; $du = \frac{1}{\ln x} \cdot \frac{1}{x}\,dx$

$$= \frac{1}{x\ln x}\,dx$$

$$\int \frac{du}{u} = \ln|u| + C = \ln|\ln(\ln(x))| + C.$$

E.g. $\left(-\frac{1}{9}\right)\int -9x^8 e^{\overset{u}{\boxed{-x^9}}}\,dx$ Let $u = -x^9$

du $du = -9x^8\,dx$

$$\rightarrow -\frac{1}{9}\int e^u\,du = -\frac{1}{9}e^u + C = \boxed{-\frac{1}{9}e^{-x^9} + C}$$