5.7. Integrals that result in Inverse Trig Functions

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Basic Derivatives of Inverse Trig Function:

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}.$$

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arccos x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\operatorname{arcsec} x) = \frac{1}{|x|\sqrt{x^2-1}}$$

Basic Integrals that result in inverse trig functions.

$$\int \frac{dx}{1+x^2} = \arctan(x) + C; \quad \int \frac{du}{1+u^2} = \arctan(u) + C$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$$

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$$\boxed{\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C.}$$

$$\boxed{\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right) + C}$$

$$\boxed{\int \frac{du}{|u|\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\left(\frac{u}{a}\right) + C}$$

E.g. $\frac{1}{4}\int \frac{\overset{du}{4dx}}{49+16x^2}$; let $u = 4x$; $du = 4dx$

$$\frac{1}{4}\int \frac{du}{49+u^2} = \frac{1}{4}\cdot\frac{1}{7}\arctan\left(\frac{u}{7}\right) + C$$

$$= \boxed{\frac{1}{28}\arctan\left(\frac{4x}{7}\right) + C}$$