① $C(x) = \dfrac{7x}{11 + x^2}$

$dC = C'(x)\, dx$.

dx = change in x = $1.43 - 1 = 0.43$

$C'(x) = \dfrac{7\cdot(11+x^2) - 7x\cdot 2x}{(11+x^2)^2}$

$= \dfrac{77 - 7x^2}{(11+x^2)^2}$

$C'(1) = \dfrac{70}{144} = \dfrac{35}{72}$

So, $dC = \dfrac{35}{72}\cdot(0.43) \approx 0.21$

② $x^3 + y^3 = 9 \xrightarrow{\frac{d}{dt}} 3x^2\cdot\dfrac{dx}{dt} + 3y^2\cdot\dfrac{dy}{dt} = 0$

Plug $x = 1$, $y = 2$, $\dfrac{dx}{dt} = -3$ into the equation:

$3\cdot(1)^2\cdot(-3) + 3\cdot(2)^2\cdot\dfrac{dy}{dt} = 0$

So, $\dfrac{dy}{dt} = \dfrac{9}{12} = \boxed{\dfrac{3}{4}}$

③ $a = 9$.

$L(x) = f'(a)(x-a) + f(a)$

$f'(x) = \dfrac{1}{2\sqrt{x}}$; $f(9) = \sqrt{9} = 3$; $f'(9) = \dfrac{1}{6}$

$\cancel{L(x) = \dfrac{1}{2\sqrt{x}}}$ $L(x) = \dfrac{1}{6}\cdot(x-9) + 3$

$L(x) = \dfrac{1}{6}x - \dfrac{3}{2} + 3$

$\boxed{L(x) = \dfrac{1}{6}x + \dfrac{3}{2}}$

④ $f'(x) = 2x + 5$

$f'(c) = \dfrac{f(1) - f(-2)}{1 - (-2)}$

$2c + 5 = \dfrac{8 - (-4)}{3} = 4$

$\boxed{c = -\dfrac{1}{2}}$

⑤ $y = 3^{\cos(\pi\theta)}$

Recall: $\dfrac{d}{dx}\left[a^u\right] = a^u\cdot\dfrac{du}{dx}\cdot(\ln a)$

So, $\dfrac{dy}{d\theta} = 3^{\cos(\pi\theta)}\cdot(-\sin(\pi\theta))\cdot\pi\cdot(\ln 3)$

⑥ $x^3 + 3x^2y + y^3 = 8 \to \dfrac{d}{dx}\left[x^3 + 3x^2y + y^3\right] = 0$

$\to 3x^2 + 6xy + 3x^2\dfrac{dy}{dx} + 3y^2\cdot\dfrac{dy}{dx} = 0$

$(3x^2 + 3y^2)\dfrac{dy}{dx} = -(3x^2 + 6xy)$

$\dfrac{dy}{dx} = -\dfrac{3x^2 + 6xy}{3x^2 + 3y^2} = -\dfrac{x^2 + 2xy}{x^2 + y^2}$

⑦ $y = (\cos x)^x \to \ln y = \ln\left[(\cos x)^x\right]$

$\to \ln y = x\cdot\ln(\cos x)$

$\to \dfrac{y'}{y} = \ln(\cos x) + x\cdot\dfrac{(-\sin x)}{\cos x}$

$\to y' = y\left[\ln(\cos x) - x\tan x\right]$

$\to y' = (\cos x)^x\left[\ln(\cos x) - x\tan x\right]$

⑧ $f'(x) = x^{1/3}(x-1)$

$f'(x) = 0 \leftrightarrow x = 0,\ x = 1$

$+ \quad 0 \quad - \quad 1 \quad +$

Increasing on $(-\infty, 0) \cup (1, \infty)$

Decreasing on $(0, 1)$

⑨ Increasing on $(-\infty, -\sqrt{2}) \cup (0, \sqrt{2})$

Decreasing on $(-\sqrt{2}, 0) \cup (\sqrt{2}, \infty)$

Concave ~~up~~ down $\left(-\dfrac{\sqrt{6}}{3}, \dfrac{\sqrt{6}}{3}\right)$

Concave ~~down~~ up $\left(-\infty, -\dfrac{\sqrt{6}}{3}\right) \cup \left(\dfrac{\sqrt{6}}{3}, \infty\right)$

$\to$ graph: B

(10) $\lim\limits_{x\to\infty} \dfrac{5x^3+4x^2}{6x^2-x} \overset{\text{behaves like}}{\approx} \dfrac{5x^3}{6x^2} = \dfrac{5}{6}x \longrightarrow \infty$

as $x \to \infty$

(11) $3x^2y - \pi\cos y = 4\pi$

$\dfrac{d}{dx}\left[3x^2y - \pi\cos y\right] = 0$

$6xy + 3x^2\dfrac{dy}{dx} - \pi\cdot(-\sin y)\cdot\dfrac{dy}{dx} = 0$

$\left(3x^2 + \pi\sin y\right)\dfrac{dy}{dx} = -6xy$

To find $\dfrac{dy}{dx}$ at $(1,\pi)$ plug $x=1$ and $y=\pi$ into the equation:

$\left(3 + \pi\sin(\pi)\right)\dfrac{dy}{dx} = -6\pi$

$3\dfrac{dy}{dx} = -6\pi \rightarrow \dfrac{dy}{dx} = -2\pi.$

Equation: $y - \pi = -2\pi\cdot(x-1)$

$y = -2\pi x + 2\pi + \pi$

$y = -2\pi x + 3\pi.$

(12) False because $f(x) = \frac{1}{x}$ is neither continuous nor differentiable at $x=0$ which is in the interval $(-1,1)$.

(13) At $x=a$, y' does not exist; neither does y''

At $x=b$, $y'=0$, $y''>0$ (concave up)

At $x=c$, $y'>0$, $y''=0$

At $x=d$, $y'=0$, $y''=0$

At $x=e$, $y'>0$, $y''=0$

At $x=f$, $y'=0$, $y''<0$ (concave down)

At $x=g$, $y'<0$, $y''=0$

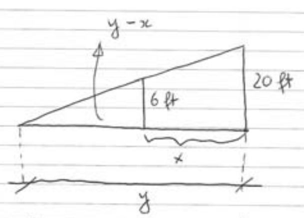
(14) $y'' = (x-5)(x+6)$

$y'' = 0$ when $x=5$; $x=-6$

$$\begin{array}{ccccc} + & & - & & + \\ \hline & -6 & & 5 & \end{array}$$

y has an inflection point at $x=-6$ and $x=5$

(15)



Given $\dfrac{dx}{dt} = 7$ ft/s. Find $\dfrac{dy}{dt}$ when $y = 35$ ft

Relation between x and y: $\dfrac{y-x}{y} = \dfrac{6}{20} = \dfrac{3}{10}$

(similar triangles)

$\longrightarrow 10(y-x) = 3y \rightarrow 7y = 10x$

Differentiate w.r.t. t: $7\dfrac{dy}{dt} = 10\dfrac{dx}{dt}$

$\rightarrow \dfrac{dy}{dt} = \dfrac{10\cdot 7}{7} = 10$ (ft/s)

(16) $y = \sqrt[4]{\dfrac{(4x+1)(x+3)^2}{(x^3+6)(x+7)}}$

$\ln y = \dfrac{1}{4}\ln\left[\dfrac{(4x+1)(x+3)^2}{(x^3+6)(x+7)}\right]$

$\ln y = \dfrac{1}{4}\left[\ln(4x+1) + 2\ln(x+3) - \ln(x^3+6) - \ln(x+7)\right]$

$\longrightarrow$ Take $\dfrac{d}{dx}$ of both sides:

$\dfrac{y'}{y} = \dfrac{1}{4}\left[\underbrace{\dfrac{4}{4x+1} + \dfrac{2}{x+3} - \dfrac{3x^2}{x^3+6} - \dfrac{1}{x+7}}\right]$

$y' = \dfrac{1}{4}y\cdot[\text{Stuff}]$

(17) $e^{xy} = \sin x \longrightarrow$ Take $\dfrac{d}{dx}$ of both sides.

$e^{xy}\cdot\dfrac{d}{dx}[xy] = \cos x$

$e^{xy}\cdot\left[y + x\dfrac{dy}{dx}\right] = \cos x$

$y + x\dfrac{dy}{dx} = \dfrac{\cos x}{e^{xy}} \longrightarrow \dfrac{dy}{dx} = \dfrac{\dfrac{\cos x}{e^{xy}} - y}{x}$

(18) $f(x) = e^x - 4x$ on $[0,2]$.

$f'(x) = e^x - 4 = 0 \longrightarrow e^x = 4$

$\longrightarrow x = \ln(4)$

$f(0) = 1$, $f(2) = e^2 - 8$

$f(\ln 4) = e^{\ln 4} - 4\ln 4 = 4 - 4\ln 4$.

The absolute min value is

(19) $V = \frac{4}{3}\pi R^3$

$dV = \frac{4}{3}\pi \cdot 3R^2 \cdot dR = 4\pi \cdot R^2 dR$

$dR = 1.2 - 1 = 0.2$.

$R = 1$

$\longrightarrow dV = 4\pi \cdot (0.2) = (0.8)\pi$.