Practice Exam 2 - Calculus I - Spring 2018

MULTIPLE CHOICE. (5pts each) Choose the one alternative that best completes the statement or answers the question. Write your answer in the space provided. No partial credit.

Solve the problem.

1) The concentration of a certain drug in the bloodstream x hr after being administered is approximately $C(x) = \frac{7x}{11 + x^2}$. Use the differential to approximate the change in concentration as x changes from 1 to 1.43. 1) _____

A) 0.34 B) 0.21 C) 0.35 D) 0.72

Provide an appropriate response.

2) If $x^3 + y^3 = 9$ and $dx/dt = -3$, then what is dy/dt when $x = 1$ and $y = 2$? 2) _____

A) $-\frac{3}{4}$ B) $\frac{4}{3}$ C) $\frac{3}{4}$ D) $-\frac{4}{3}$

You want a linearization that will replace the function over an interval that includes the point x_0. To make your subsequent work as simple as possible, you want to center the linearization not at x_0 but at a nearby integer $x = a$ at which the function and its derivative are easy to evaluate. What linearization do you use?

3) $f(x) = \sqrt{x}$, $x_0 = 9.1$ 3) _____

A) $\frac{3}{2} + \frac{1}{6}x$ B) $\frac{3}{2} + \frac{1}{3}x$ C) $\frac{3}{2} - \frac{1}{6}x$ D) $\frac{1}{2} + \frac{1}{2}x$

Find the value or values of c that satisfy the equation $\frac{f(b) - f(a)}{b - a} = f'(c)$ in the conclusion of the Mean Value Theorem for the function and interval.

4) $f(x) = x^2 + 5x + 2$, $[-2, 1]$ 4) _____

A) $-\frac{1}{2}$ B) $-2, 1$ C) $-\frac{1}{2}, \frac{1}{2}$ D) $0, -\frac{1}{2}$

Find the derivative of y with respect to the independent variable.

5) $y = 3^{\cos \pi\theta}$ 5) _____

A) $3^{\cos \pi\theta}$ B) $-\pi 3^{\cos \pi\theta} \ln 3 \sin \pi\theta$

C) $\pi 3^{\cos \pi\theta} \ln 3$ D) $-3^{\cos \pi\theta} \ln 3 \sin \pi\theta$

Use implicit differentiation to find dy/dx.

6) $x^3 + 3x^2y + y^3 = 8$ 6) _____

A) $\frac{x^2 + 3xy}{x^2 + y^2}$ B) $\frac{x^2 + 2xy}{x^2 + y^2}$ C) $-\frac{x^2 + 2xy}{x^2 + y^2}$ D) $-\frac{x^2 + 3xy}{x^2 + y^2}$

Use logarithmic differentiation to find the derivative of y with respect to the independent variable.

7) $y = (\cos x)^x$ 7) _____

A) $\ln \cos x - x \tan x$ B) $\ln x(\cos x)^{x - 1}$

C) $(\cos x)^x (\ln \cos x + x \cot x)$ D) $(\cos x)^x (\ln \cos x - x \tan x)$

Using the derivative of f(x) given below, determine the intervals on which f(x) is increasing or decreasing.

8) $f'(x) = x^{1/3}(x - 1)$ 8) _____

A) Decreasing on $(0, 1)$; increasing on $(-\infty, 0) \cup (1, \infty)$

B) Decreasing on $(-\infty, 0) \cup (1, \infty)$; increasing on $(0, 1)$

C) Decreasing on $(0, 1)$; increasing on $(1, \infty)$

D) Increasing on $(0, \infty)$

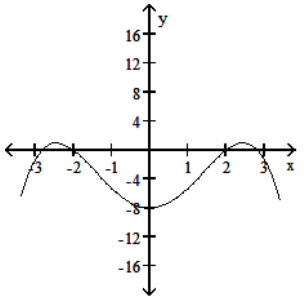
Solve the problem.

9) Select an appropriate graph of a twice-differentiable function $y = f(x)$ that passes through the points $(-\sqrt{2},1)$, $\left(-\frac{\sqrt{6}}{3},\frac{5}{9}\right)$, $(0,0)$, $\left(\frac{\sqrt{6}}{3},\frac{5}{9}\right)$ and $(\sqrt{2},1)$, and whose first two derivatives have the following sign patterns. 9) _____

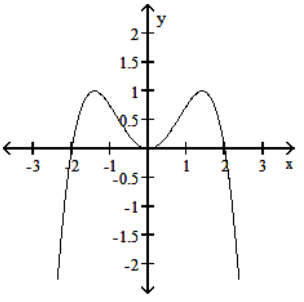
y' : $\underset{-\sqrt{2}}{+} \quad \underset{0}{-} \quad \underset{\sqrt{2}}{+} \quad -$

y'' : $+ \quad - \quad +$ over $-\frac{\sqrt{6}}{3}$ and $\frac{\sqrt{6}}{3}$

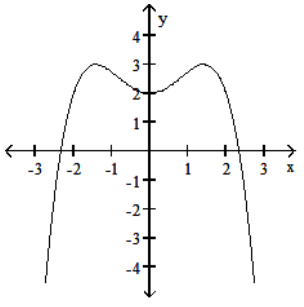
A)



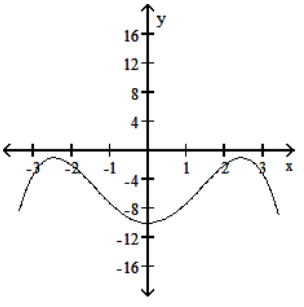
B)



C)



D)



Find the limit, if it exists.

10) $\lim\limits_{x \to \infty} \frac{5x^3 + 4x^2}{6x^2 - x}$ 10) _____

A) 0 B) 5 C) ∞ D) $\frac{2}{3}$

SHORT ANSWER. (5pts each) Write the word or phrase that best completes each statement or answers the question. Write your answer in the space provided. No partial credit.

At the given point, find the slope of the curve, the line that is tangent to the curve, or the line that is normal to the curve, as requested.

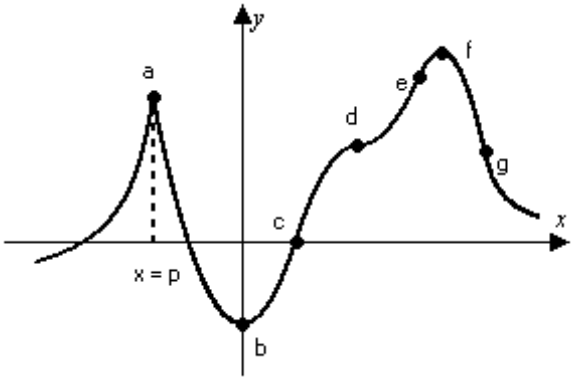
11) $3x^2y - \pi \cos y = 4\pi$, slope at $(1, \pi)$ 11) _____

Provide an appropriate response.

12) Decide if the statement is true or false. If false, explain. 12) _____

The points $(-1, -1)$ and $(1, 1)$ lie on the graph of $f(x) = \frac{1}{x}$. Therefore, the Mean Value Theorem says that there exists some value $x = c$ on $(-1, 1)$ for which $f'(x) = \frac{1 - (-1)}{1 - (-1)} = 1$.

13) The accompanying figure shows a portion of the graph of a function that is twice-differentiable at all x except at x = p. At each of the labeled points, classify y′ and y″ as positive, negative, or zero. 13) _____



14) Suppose that the second derivative of the function y = f(x) is y" = (x - 5)(x + 6). For what x-values does the graph of f have an inflection point? 14) _____

ESSAY.(6pts each) Write your answer in the space provided or on a separate sheet of paper. Show all work. Answers with no work or insufficient work will receive no credit. Partial credit may be given for correct work.

Solve the problem. Round your answer, if appropriate.

15) A man 6 ft tall walks at a rate of 7 ft/sec away from a lamppost that is 20 ft high. At what rate is the length of his shadow changing when he is 35 ft away from the lamppost? (Do not round your answer)

Use logarithmic differentiation to find the derivative of y.

16) $y = \sqrt[4]{\dfrac{(4x + 1)(x + 3)^2}{(x^3 + 6)(x + 7)}}$

Find $\frac{dy}{dx}$.

17) $e^{xy} = \sin x$

Answer the question appropriately.

18) Find the absolute minimum value of $f(x) = e^x - 4x$ on $[0, 2]$.

Solve the problem.

19) $V = \frac{4}{3}\pi r^3$, where r is the radius, in centimeters. Use differential toa approximate how much does the volume of a sphere increase when the radius is increased from 1.0 cm to 1.2 cm? (Use 3.14 for π.)

1) B
2) C
3) A
4) A
5) B
6) C
7) D
8) A
9) B
10) C
11) -2π
12) False. The function has a non-removable discontinuity at $x = 0$. The mean value theorem does not apply.
13) a: both y' and y'' are undefined.
    b: $y' = 0$ and $y'' > 0$
    c: $y' > 0$ and $y'' = 0$
    d: $y' = 0$ and $y'' = 0$
    e: $y' > 0$ and $y'' = 0$
    f: $y' = 0$ and $y'' < 0$
    g: $y' < 0$ and $y'' = 0$
14) 5, -6
15) 3 ft/sec
16) $\frac{1}{4}\sqrt[4]{\frac{(4x+1)(x+3)^2}{(x^3+6)(x+7)}}\left(\frac{4}{4x+1}+\frac{2}{x+3}-\frac{3x^2}{x^3+6}-\frac{1}{x+7}\right)$
17) $\frac{\cos x - ye^{xy}}{xe^{xy}}$
18) $4 - 4 \ln 4$
19) $2.5\ \text{cm}^3$