Practice Exam 3 - Calculus I - Spring 2018

MULTIPLE CHOICE. (5pts each) Choose the one alternative that best completes the statement or answers the question. Write your answer in the space provided. No partial credit.

Evaluate the limit.

1) $\lim\limits_{x \to \infty} \dfrac{2 + 5x - 16x^2}{16 + 3x - 19x^2}$ 1) ________

A) $\dfrac{1}{8}$ B) $\dfrac{16}{19}$ C) 1 D) ∞

Use l'Hôpital's rule to find the limit.

2) $\lim\limits_{x \to \infty} x \sin \dfrac{10}{x}$ 2) ________

A) 0 B) 1 C) $\dfrac{1}{10}$ D) 10

Find the most general antiderivative.

3) $\displaystyle\int \left(\frac{\sqrt{y}}{5} + \frac{8}{\sqrt{y}}\right) dy$ 3) ________

A) $\dfrac{2}{15}y^{3/2} + 16\sqrt{y} + C$ B) $\dfrac{1}{10}\sqrt{y} - \dfrac{1}{16\sqrt{y}} + C$

C) $\dfrac{2}{15}y^{3/2} - 16\sqrt{y} + C$ D) $\dfrac{3}{10}y^{3/2} + \dfrac{1}{16}\sqrt{y} + C$

4) $\displaystyle\int \sin \theta(\cot \theta + \csc \theta)\, d\theta$ 4) ________

A) $\cos \theta + C$ B) $\sin \theta + C$

C) $\csc \theta + \cos \theta + C$ D) $\sin \theta + \theta + C$

Solve the initial value problem.

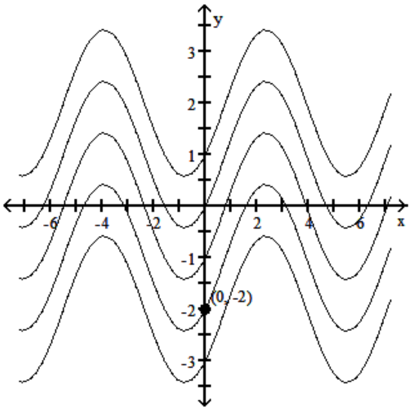
5) $\dfrac{d^2r}{dt^2} = \dfrac{4}{t^3}$; $\left.\dfrac{dr}{dt}\right|_{t=1} = 2$, $r(1) = 5$ 5) ________

A) $r = \dfrac{2}{t} + 4t + 11$ B) $r = 2t + 4t + 11$

C) $r = \dfrac{4}{-5t^5} + \dfrac{14}{5}t - 1$ D) $r = \dfrac{2}{t} + 4t - 1$

The graph below shows solution curves of a differential equation. Find an equation for the curve through the given point.

6) 6) ________



$$\frac{dy}{dx} = \sin x + \cos x$$

A) $y = \sin x - \cos x - 2$
B) $y = \sin x - \cos x - 1$
C) $y = \sin x - \cos x$
D) $y = \sin x - \cos x + 1$

Use a finite approximation to estimate the area under the graph of the given function on the stated interval as instructed.

7) $f(x) = x^2$ between $x = 1$ and $x = 5$ using a lower sum with four rectangles of equal width. 7) ________

A) 69
B) 41
C) 54
D) 30

Solve the problem.

8) Suppose that f is continuous and that $\int_{-2}^{2} f(z)\,dz = 0$ and $\int_{-2}^{7} f(z)\,dz = 4$. Find $\int_{7}^{2} f(x)\,dx$. 8) ________

A) 4
B) 8
C) -4
D) -8

Graph the integrand and use areas to evaluate the integral.

9) $\int_{-7}^{7} \sqrt{49 - x^2}\,dx$ 9) ________

A) 7π
B) $\frac{49}{2}\pi$
C) 49
D) 49π

Find the derivative.

10) $\frac{d}{dt}\int_{0}^{\sin t} \frac{1}{9 - u^2}\,du$ 10) ________

A) $\frac{1}{9 - \sin^2 t}$
B) $\frac{1}{\cos t\,(9 - \sin^2 t)}$
C) $\frac{-\cos t}{9 - \sin^2 t}$
D) $\frac{\cos t}{9 - \sin^2 t}$

SHORT ANSWER. (5pts each) Write the word or phrase that best completes each statement or answers the question. Write your answer in the space provided. No partial credit.

Solve the problem.

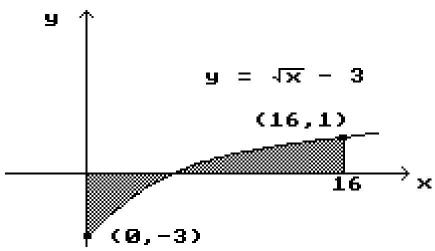
11) Use Newton's method to estimate the one real solution of the equation $3x^5 - 2x - 4 = 0$. Start with $x_1 = 1$. Then find x_2. 11) ______________

Evaluate the integral.

12) $\int_1^2 \left(t + \frac{1}{t}\right)^2 dt$ 12) ______________

Find the area of the shaded region.

13) 13) ______________



Provide an appropriate response.

14) What definite integral is represented by $\lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{4i}{n}\right)^2 \frac{4}{n}$? Use the Fundamental Theorem to evaluate the integral. 14) ______________

ESSAY. (6pts each) Write your answer in the space provided or on a separate sheet of paper. Show all work. Answers with no work or insufficient work will receive no credit. Partial credit may be given for correct work.

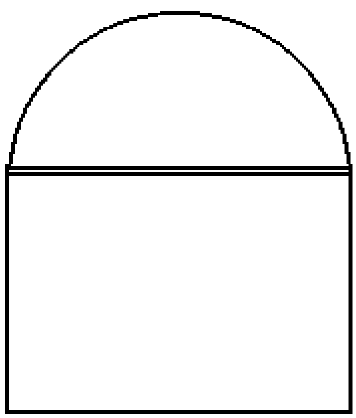
15) Find the linearization of $f(x) = 3 + \int_1^{x+1} \tan \frac{\pi t}{4} \, dt$ at $x = 0$.

Evaluate the integral.

16) $\int_1^4 \frac{t^2 + 1}{\sqrt{t}} \, dt$

Solve the problem.

17) A window is in the form of a rectangle surmounted by a semicircle. The rectangle is of clear glass, whereas the semicircle is of tinted glass that transmits only one-fifth as much light per unit area as clear glass does. The total perimeter is fixed. Find the proportions of the window that will admit the most light. Neglect the thickness of the frame.



Find the limit.

18) $\lim_{x \to \infty} \left(1 + \frac{2}{x^5}\right)^x$

Find the formula and limit as requested.

19) For the function $f(x) = 9 - 5x^2$, find a formula for the lower sum obtained by dividing the interval [0, 1] into n equal subintervals. Then take the limit as $n \to \infty$ to calculate the area under the curve over [0, 1].