① $\displaystyle\int_0^{\pi/8} \frac{\sec^2(2x)}{2+\tan(2x)}\,dx$

Let $u = 2+\tan(2x)$

Then $\boxed{du = 2\sec^2(2x)\,dx}$ (Chain Rule)

Rewrite the original integral as:

$$\frac{1}{2}\int_0^{\pi/8} \frac{\boxed{2\sec^2(2x)\,dx}^{\,du}}{\boxed{2+\tan(2x)}_{\,u}}$$

Change the bounds:

When $x=0$: $u = 2+\tan(2\cdot 0) = 2+\tan(0) = 2$

When $x=\frac{\pi}{8}$: $u = 2+\tan\left(2\cdot\frac{\pi}{8}\right) = 2+\tan\left(\frac{\pi}{4}\right) = 3$

So, the integral in terms of u is:

$$\frac{1}{2}\int_2^3 \frac{du}{u} = \frac{1}{2}\ln|u|\Big|_2^3 = \frac{1}{2}\left[\ln(3) - \ln(2)\right]$$

$$= \boxed{\frac{1}{2}\ln\left(\frac{3}{2}\right)}$$

② $\displaystyle\int \frac{e^{1/x}}{5x^2}\,dx = \frac{1}{5}\int \frac{e^{1/x}}{x^2}\,dx$

Let $u = \frac{1}{x}$. Then $\boxed{du = -\frac{1}{x^2}\,dx}$

Rewrite the original integral as:

$$-\frac{1}{5}\int e^{\overset{u}{1/x}}\,\boxed{\frac{dx}{-x^2}}^{\,du}$$

So, it becomes:

$$-\frac{1}{5}\int e^u\,du = -\frac{1}{5}e^u + C.$$

So, the answer is

$$\boxed{-\frac{1}{5}e^{1/x} + C}$$

$I_2 = \displaystyle\int \frac{12x}{9+36x^2}\,dx$.

Let $u = 9+36x^2$. Then $du = 72x\,dx$.

We have:

$$I_2 = \frac{1}{6}\int \frac{\overset{du}{72x\,dx}}{\boxed{9+36x^2}_{\,u}}$$

$$= \frac{1}{6}\int \frac{du}{u} = \frac{1}{6}\ln|u| + C$$

$$= \frac{1}{6}\ln|9+36x^2| + C.$$

So, the answer is:

$$\frac{1}{3}\arctan(2x) + \frac{1}{6}\ln|9+36x^2| + C$$

③ $\displaystyle\int \frac{6+12x}{9+36x^2}\,dx$

$$= \underbrace{\int \frac{6}{9+36x^2}\,dx}_{I_1} + \underbrace{\int \frac{12x}{9+36x^2}\,dx}_{I_2}$$

$$I_1 = \int \frac{6}{9+36x^2}\,dx = \int \frac{\boxed{6\,dx}^{\,du}}{9+(\boxed{6x}_{\,u})^2}$$

Let $u = 6x$. Then $du = 6\,dx$. So it becomes:

$$\int \frac{du}{9+u^2} = \frac{1}{3}\arctan\left(\frac{u}{3}\right) + C$$

$$= \frac{1}{3}\arctan\left(\frac{6x}{3}\right) + C$$

$$= \frac{1}{3}\arctan(2x) + C$$

(4) $\text{Area} = \int_a^b (\text{top curve} - \text{bottom curve})\, dx$

$$= \int_0^1 \left[-x^4 - (x^2 - 2x)\right] dx$$

$$= \int_0^1 (-x^4 - x^2 + 2x)\, dx$$

$$= \left(-\frac{x^5}{5} - \frac{x^3}{3} + x^2\right)\Bigg|_0^1$$

$$= -\frac{1}{5} - \frac{1}{3} + 1 = \boxed{\frac{7}{15}}$$

(5) Points of Intersection:

$$2 = 2\sin(\pi x) \longleftrightarrow \sin(\pi x) = 1$$

$$\longleftrightarrow \pi x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots \longrightarrow x = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$$

from picture

$$\text{Area} = \int_{1/2}^{5/2} \left[2 - 2\sin(\pi x)\right] dx$$

$$= \left[2x + \frac{2}{\pi}\cos(\pi x)\right]\Bigg|_{1/2}^{5/2}$$

$$= \left[2\cdot\frac{5}{2} + \frac{2}{\pi}\cos\left(\frac{3\pi}{2}\right)\right] - \left[2\cdot\frac{1}{2} + \frac{2}{\pi}\cos\left(\frac{\pi}{2}\right)\right]$$

$$= 5 - 1 = \boxed{4}$$

(6) $y = \dfrac{4}{x^9} - \dfrac{2x^4}{x^9} + \dfrac{x^5}{x^9}$

$$= 4\cdot x^{-9} - 2x^{-5} + x^{-4}$$

$$\frac{dy}{dx} = -36x^{-10} + 10x^{-6} - 4x^{-5}$$

$$= \boxed{-\frac{36}{x^{10}} + \frac{10}{x^6} - \frac{4}{x^5}}$$

(7) $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{u'v - uv'}{v^2}$

So $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)\Big|_{\text{at } x=2} = \dfrac{u'(2)\cdot v(2) - u(2)v'(2)}{[v(2)]^2}$

$$= \frac{4\cdot(-3) - 6\cdot(-5)}{[-3]^2} = \frac{-42}{9} = \boxed{\frac{-14}{3}}$$

(8) $x^3 + 3x^2y + y^3 = 8$

$$\frac{d}{dx}(x^3) + 3\cdot\frac{d}{dx}(x^2y) + \frac{d}{dx}(y^3) = 0$$

$$3x^2 + 3\cdot\left[2xy + x^2\cdot\frac{dy}{dx}\right] + 3y^2\cdot\frac{dy}{dx} = 0$$

$$3x^2 + 6xy + 3x^2\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$

$$3(x^2+y^2)\frac{dy}{dx} = -3x(x+2y)$$

$$\frac{dy}{dx} = \boxed{-\frac{x(x+2y)}{x^2+y^2} = \frac{-(x^2+2xy)}{x^2+y^2}}$$

(9) $f'(x) = 0$ when $x = 0$ or $x = 1$.

$f'(x)$ is always defined so we don't have to worry about it being undefined

So, f is increasing on $(-\infty, 0) \cup (1, \infty)$

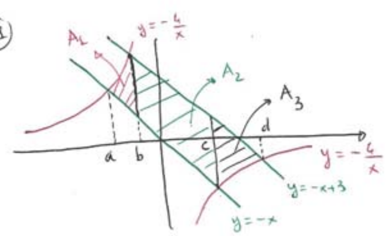
decreasing on $(0, 1)$

(10) $$\frac{d}{dt}\left(\int_0^{\sin t} \frac{1}{9-u^2}\, du \right)$$

$$= \frac{1}{9-(\sin t)^2} \cdot (\cos t)$$

$$= \boxed{\frac{\cos t}{9 - \sin^2 t}}$$

(11)



Area $= A_1 + A_2 + A_3$

To find points a and c: set $-\frac{4}{x} = -x \Leftrightarrow x^2 = 4$

$\Leftrightarrow x = \pm 2$.

So, $a = -2$; $c = 2$.

To find points b and d: Set $-\frac{4}{x} = -x+3$

$\Leftrightarrow -4 = -x^2 + 3x$

$\Leftrightarrow x^2 - 3x - 4 = 0$

$\Leftrightarrow (x-4)(x+1) = 0$

$\Leftrightarrow x = 4$; $x = -1$.

$b = -1$; $d = 4$

So, $$A_1 = \int_{-2}^{-1} \Big[\underbrace{-\frac{4}{x}}_{\text{top}} - \underbrace{(-x)}_{\text{bottom}} \Big] dx$$

$$A_2 = \int_{-1}^{2} \Big[\underbrace{-x+3}_{\text{top}} - \underbrace{(-x)}_{\text{bottom}} \Big] dx$$

$$A_3 = \int_{2}^{4} \Big[\underbrace{-x+3}_{\text{top}} - \underbrace{\left(-\frac{4}{x}\right)}_{\text{bottom}} \Big] dx$$

So, area is given by:

$$\int_{-2}^{-1} \left(-\frac{4}{x} + x\right) dx + \int_{-1}^{2} (-x+3+x)\, dx + \int_{2}^{4} \left(-x+3+\frac{4}{x}\right) dx$$

$$= \int_{-2}^{-1} \left(-\frac{4}{x} + x\right) dx + \int_{-1}^{3} 3\, dx + \int_{2}^{4} \left(-x+3+\frac{4}{x}\right) dx$$

(12) $$\int_0^{\sqrt{3}/2} \frac{(\overbrace{\sin^{-1} x}^{u})^2}{\sqrt{1-x^2}}\, dx$$ → du

Let $u = \sin^{-1} x$, then $du = \frac{1}{\sqrt{1-x^2}}\, dx$

Change of bounds:

when $x = 0$, $u = \sin^{-1}(0) = 0$

when $x = \sqrt{3}/2$; $u = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

So, the integral becomes:

$$\int_0^{\pi/3} u^2\, du = \frac{u^3}{3} \Big|_0^{\pi/3} = \frac{1}{3} \cdot \left(\frac{\pi}{3}\right)^3$$

$$= \boxed{\frac{\pi^3}{81}}$$

(13) a: y', y'' are both undefined

b: $y' = 0$, $y'' > 0$

c: $y' > 0$, $y'' = 0$

d: $y' = 0$; $y'' = 0$

e: $y' > 0$, $y'' = 0$

f: $y' = 0$; $y'' < 0$

g: $y' < 0$, $y'' = 0$

(14) 998 divided by 4 ; remainder = 2

So, $\dfrac{d^{998}}{dx^{998}}(\cos x) = \cancel{\cos x}\ -\cos x$.

(15) $\displaystyle\int \frac{dx}{x(5+8\ln x)}$ let $u = 5 + 8\ln x$. Then $du = \dfrac{8}{x}\,dx$

Rewrite:

$$\frac{1}{8}\int \frac{\overbrace{8dx}^{du}}{x(\underbrace{\boxed{5+8\ln x}}_{u})}$$

Integral becomes:

$$\frac{1}{8}\int \frac{du}{u} = \frac{1}{8}\ln|u| + C$$

$$= \frac{1}{8}\ln|5 + 8\ln x| + C.$$

(16) $$\int_0^{\sqrt{\ln\pi}} \overbrace{\boxed{2xe^{x^2}}}^{du} \sin(\underbrace{\boxed{e^{x^2}}}_{u}) \boxed{dx}$$

let $u = e^{x^2}$. Then $du = 2xe^{x^2}dx$ (Chain Rule)

Change of bounds:

when $x = 0$; $u = e^0 = 1$.

When $x = \sqrt{\ln\pi}$, $u = e^{(\sqrt{\ln\pi})^2} = e^{\ln\pi} = \pi$.

So, integral becomes:

$$\int_1^{\pi} \sin(u)\,du = -\cos(u)\Big|_1^{\pi}$$

$$= -\cos(\pi) + \cos(1)$$

$$= \boxed{1 + \cos(1)}$$

(17) $$\int_1^4 \frac{t^2+1}{\sqrt{t}}\,dt = \int_1^4 \left(\frac{t^2}{\sqrt{t}} + \frac{1}{\sqrt{t}}\right)dt$$

$$= \int_1^4 \left(\frac{t^2}{t^{1/2}} + \frac{1}{t^{1/2}}\right)dt = \int_1^4 \left(t^{3/2} + t^{-1/2}\right)dt$$

$$= \left(\frac{2t^{5/2}}{5} + 2t^{1/2}\right)\Big|_1^4$$

$$= \left[\frac{2}{5}\cdot(4)^{5/2} + 2\cdot(4)^{1/2} - \left(\frac{2}{5} + 2\right)\right]$$

$$= \frac{64}{5} + 4 - \frac{12}{5} = \boxed{\frac{72}{5}}$$

(18) Formula for linearization at a of f:

$$L(x) = f(a) + f'(a)(x-a)$$

Here $a = 0$; $f(x) = 3 + \displaystyle\int_1^{x+1} \tan\left(\frac{\pi t}{4}\right)dt$

So, $L(x) = f(0) + f'(0)\cdot x$

We need to find $f(0)$ and $f'(0)$

$$f(0) = 3 + \underbrace{\int_1^1 \tan\left(\frac{\pi t}{4}\right)dt}_{=0} = 3$$

$$f'(0) = \frac{d}{dx}\left(3 + \int_1^{x+1} \tan\left(\frac{\pi t}{4}\right)dt\right)\Bigg|_{x=0}$$

$$= \tan\left(\frac{\pi(x+1)}{4}\right)\Bigg|_{x=0} \quad \text{(FTC-I)}$$

$$= \tan\left(\frac{\pi}{4}\right) = 1.$$

So, $\boxed{L(x) = 3 + x}$

(19) $e^{xy} = \sin x$

$\longrightarrow \frac{d}{dx}\left[e^{xy}\right] = \frac{d}{dx}\left[\sin x\right]$

$e^{xy} \cdot \frac{d}{dx}\left[xy\right] = \cos x$

$e^{xy} \cdot \left[y + x\frac{dy}{dx}\right] = \cos x$

$y + x\frac{dy}{dx} = \frac{\cos x}{e^{xy}}$

$x\frac{dy}{dx} = \frac{\cos x}{e^{xy}} - y$

$$\frac{dy}{dx} = \frac{\frac{\cos x}{e^{xy}} - y}{x}$$

$$= \frac{\cos x - ye^{xy}}{xe^{xy}}$$