# 2.2. Volume by Slicing

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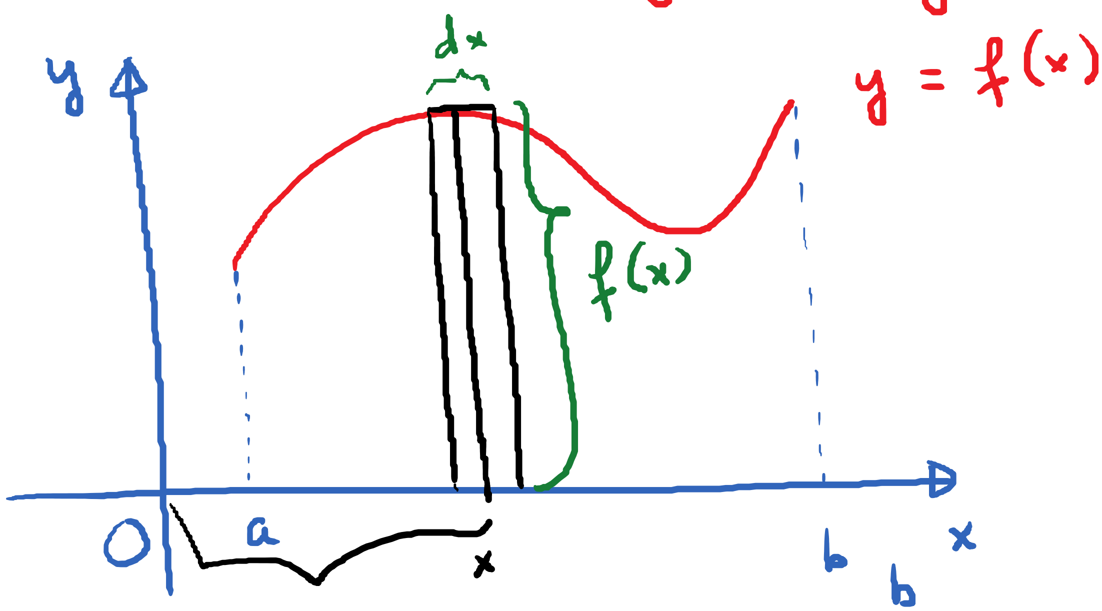
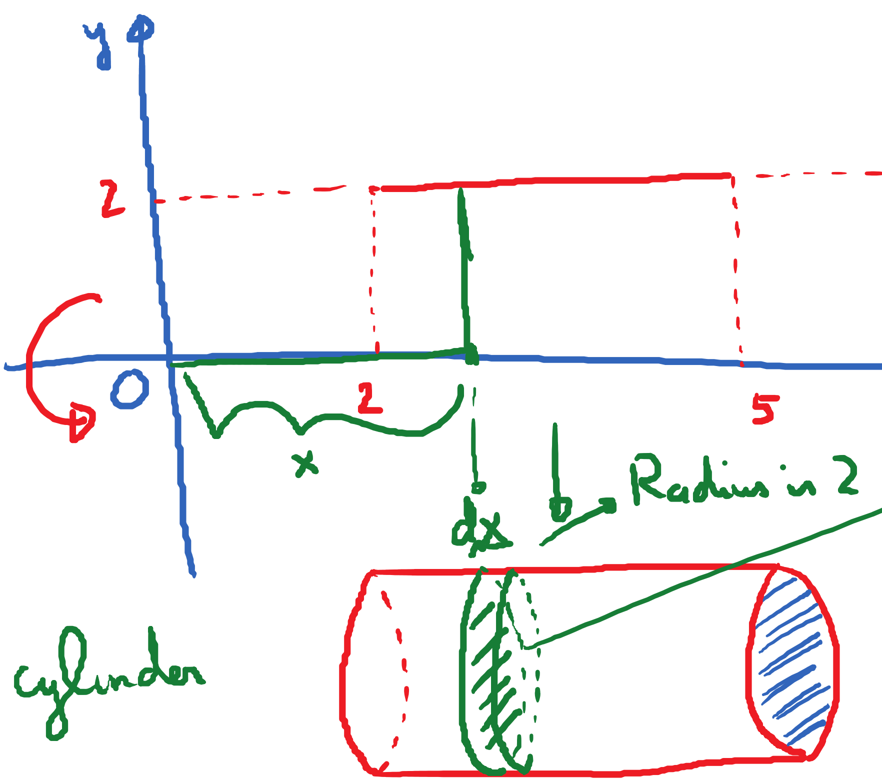



Area under $y = f(x)$, $a \leq x \leq b$ $= \int_a^b f(x)\,dx$

height · width → area of 1 slice




Area of Slice $= \pi \cdot (2)^2 = 4\pi$

Volume $= 12\pi$

$\int^{5} 4\pi\,dx$

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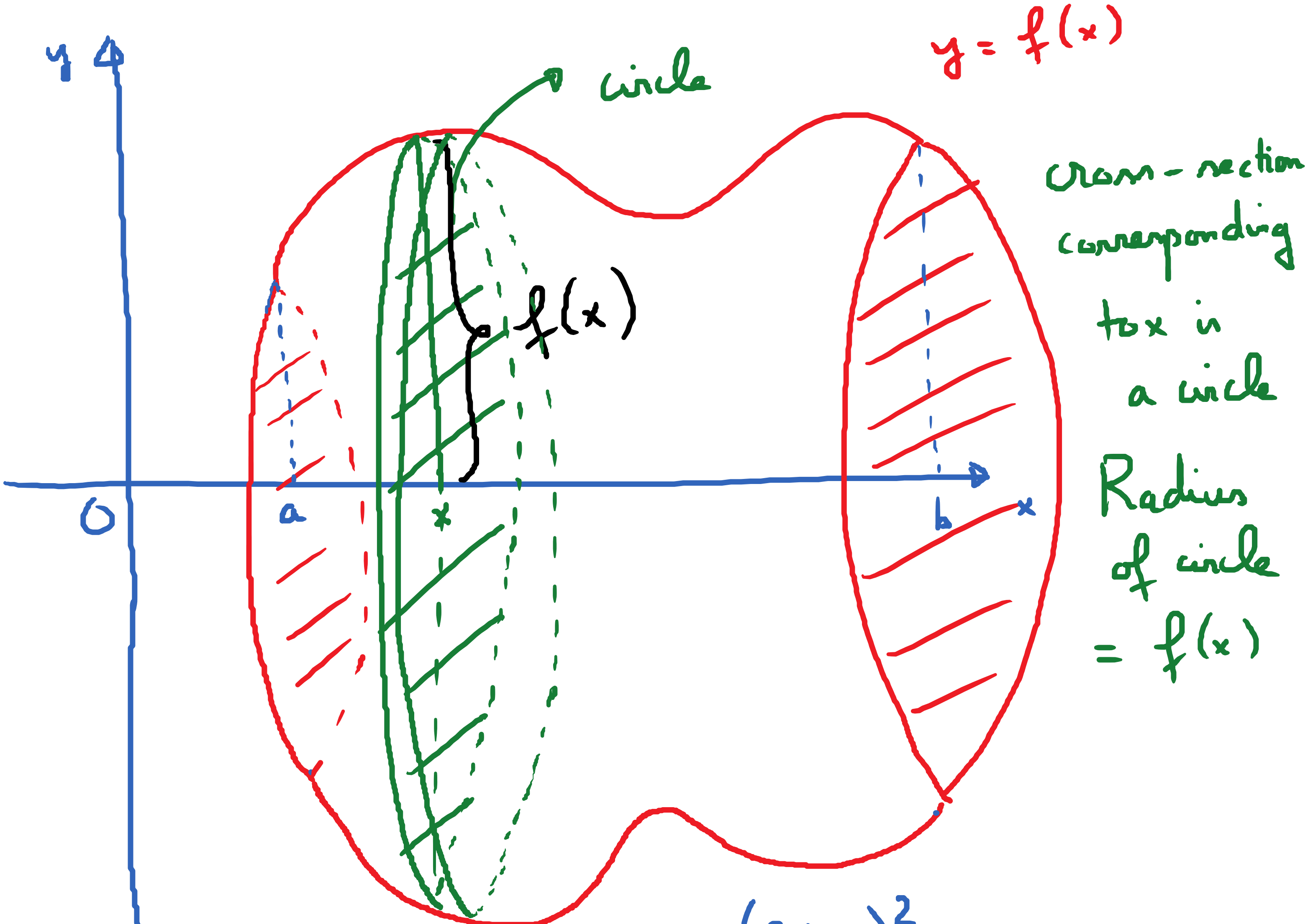
$$\int_2^5 4\pi \, dx = 4\pi \cdot \int_2^5 dx = 4\pi \cdot x \Big|_2^5$$

$$= 4\pi \cdot (5 - 2)$$

$$= 12\pi$$




Area of circle $= \pi \cdot \left(f(x)\right)^2$

Volume of slice $= \pi \left(f(x)\right)^2 \cdot dx$

Volume of object: $= \int_a^b \pi \left(f(x)\right)^2 dx$

$$= \pi \int_a^b \left(f(x)\right)^2 dx$$

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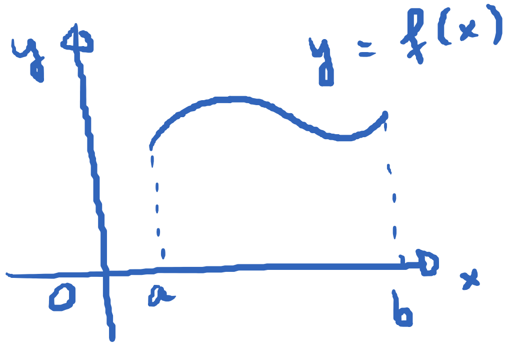
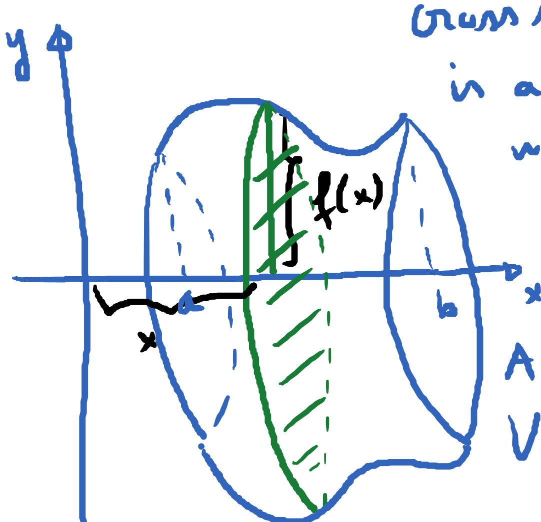
## The disk method to find volume of revolution.




Cross section at x is a circle with radius f(x)

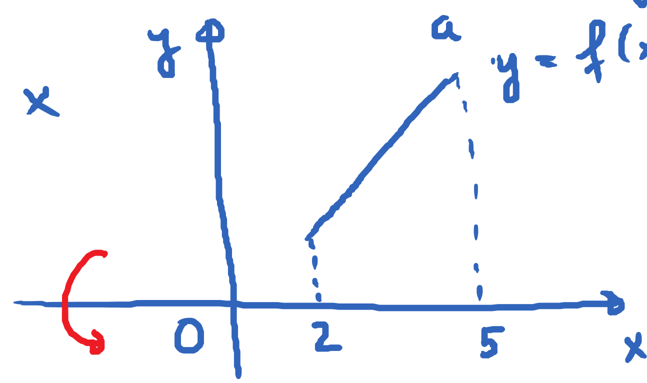
$A = \pi \left(f(x)\right)^2$

$V_{slice} = A \cdot dx$

Revolve $y = f(x)$; $a \le x \le b$ about x-axis.

Volume of object obtained $= \pi \cdot \int_a^b \left(f(x)\right)^2 dx$

E.g. $y = f(x) = x$



Rotate the region bounded by the graph of $y = x$ from $x = 2$ to $x = 5$ about the x-axis.

Obtain a solid S.

Find the volume of S.

$$V = \pi \int_2^5 x^2\,dx = \pi \cdot \frac{x^3}{3}\Big|_2^5 = \frac{\pi}{3}\cdot\left((5)^3 - (2)^3\right)$$

$$= \frac{\pi}{3}\cdot(125 - 8) = \frac{117\pi}{3} = \boxed{39\pi}$$

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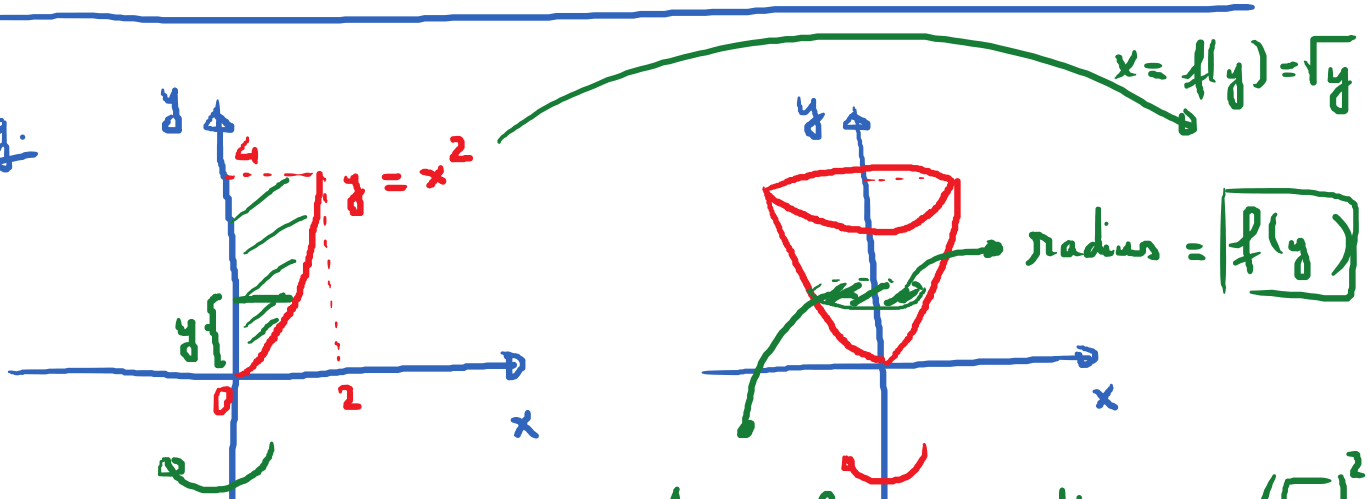
E.g.




$$V = \pi \int_0^4 (\sqrt{y})^2\,dy$$

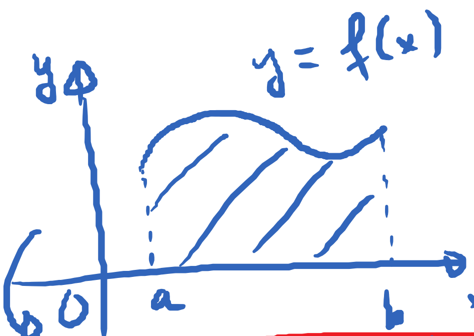
Area of cross section $= \pi \cdot (\sqrt{y})^2$

$$\underbrace{\pi \cdot (\sqrt{y})^2\,dy}_{V_{slice}}$$

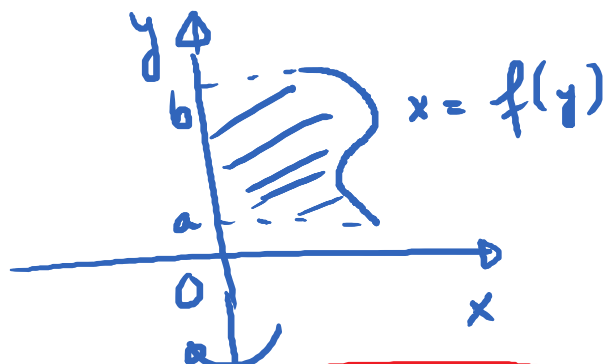
$$V = \pi \cdot \int_0^4 (\sqrt{y})^2\,dy = \pi \cdot \int_0^4 y\,dy$$

$$= \pi \cdot \frac{y^2}{2}\Bigg|_0^4 = \boxed{8\pi}.$$

# Disk Method for finding volume.




$$\boxed{\text{Volume} = \pi \cdot \int_a^b (f(x))^2\,dx}$$




$$\boxed{\text{Volume} = \pi \cdot \int_a^b (f(y))^2\,dy.}$$

(Note: in many situations, we need to solve for x in terms of y to get the formula for f(y))