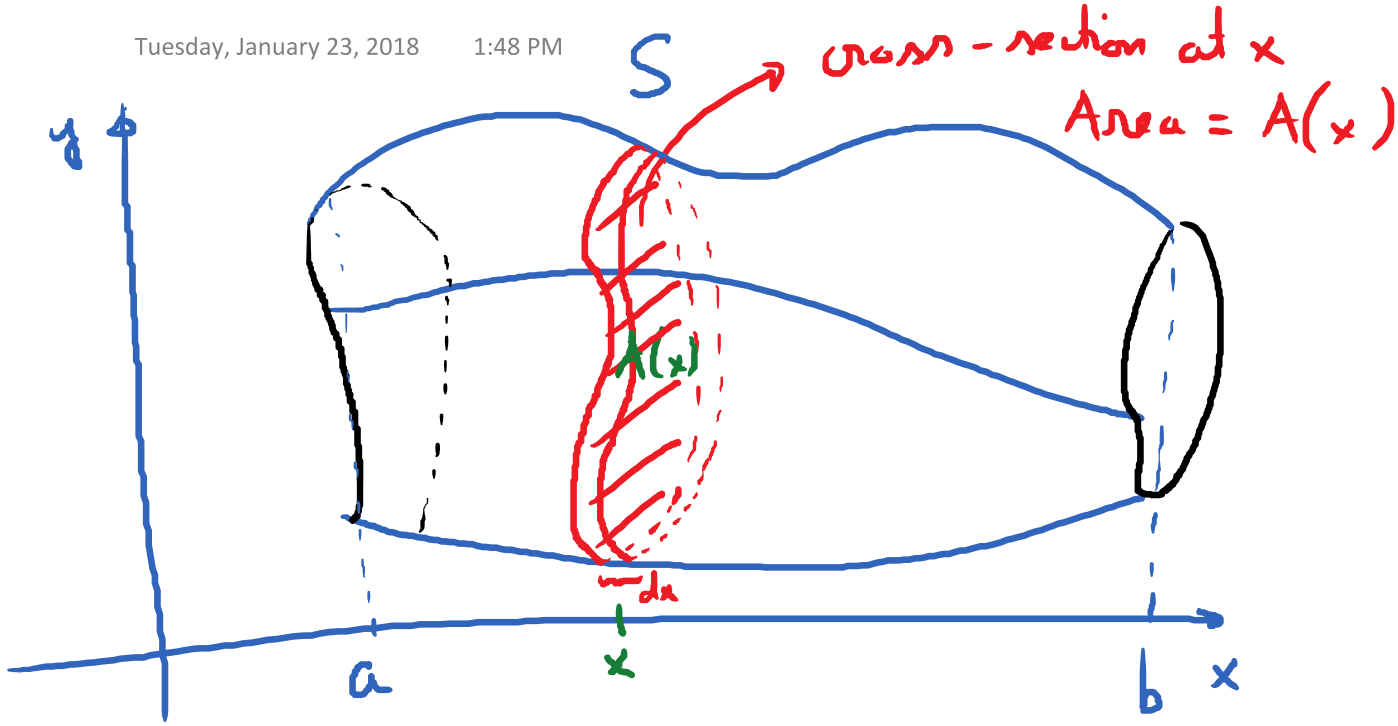


Volume of S =

If we can find a function A(x) that gives us the area of the cross section at x.

(A(x) = area) Volume of a slice

$$V = \int_a^b \boxed{\underbrace{A(x)}_{\text{cross-sectional area}} \cdot \underbrace{dx}_{\text{thickness}}}$$

HW 2 #8

Base of object: region under parabola $y = 1 - x^2$ and above x-axis

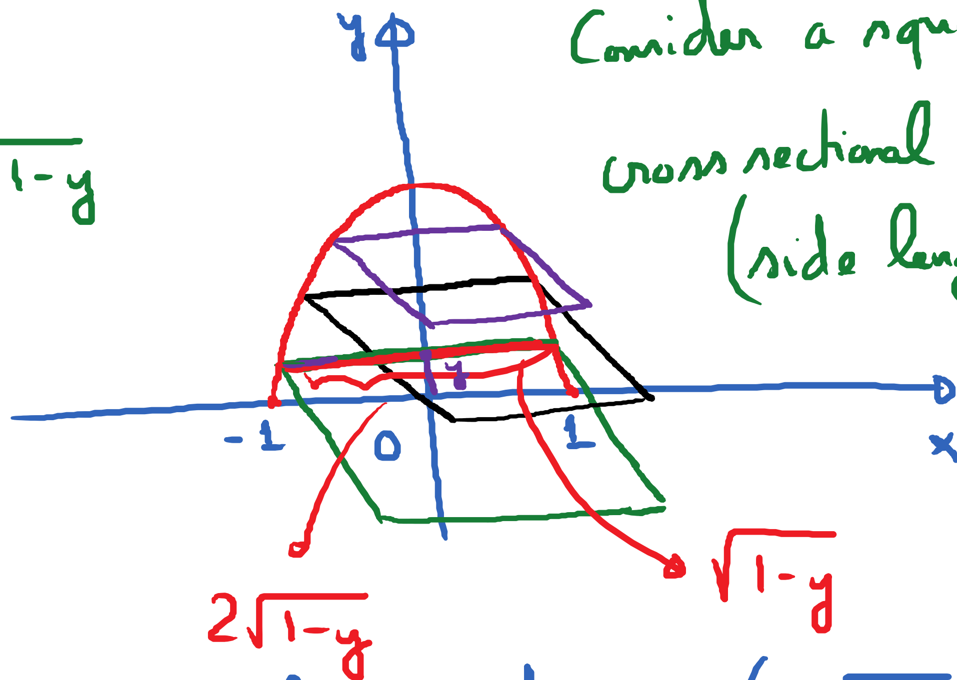
Cross sections are squares

Consider a square at y.

cross sectional area = ? (side length)^2

$x^2 = 1 - y$

$x = \pm\sqrt{1-y}$



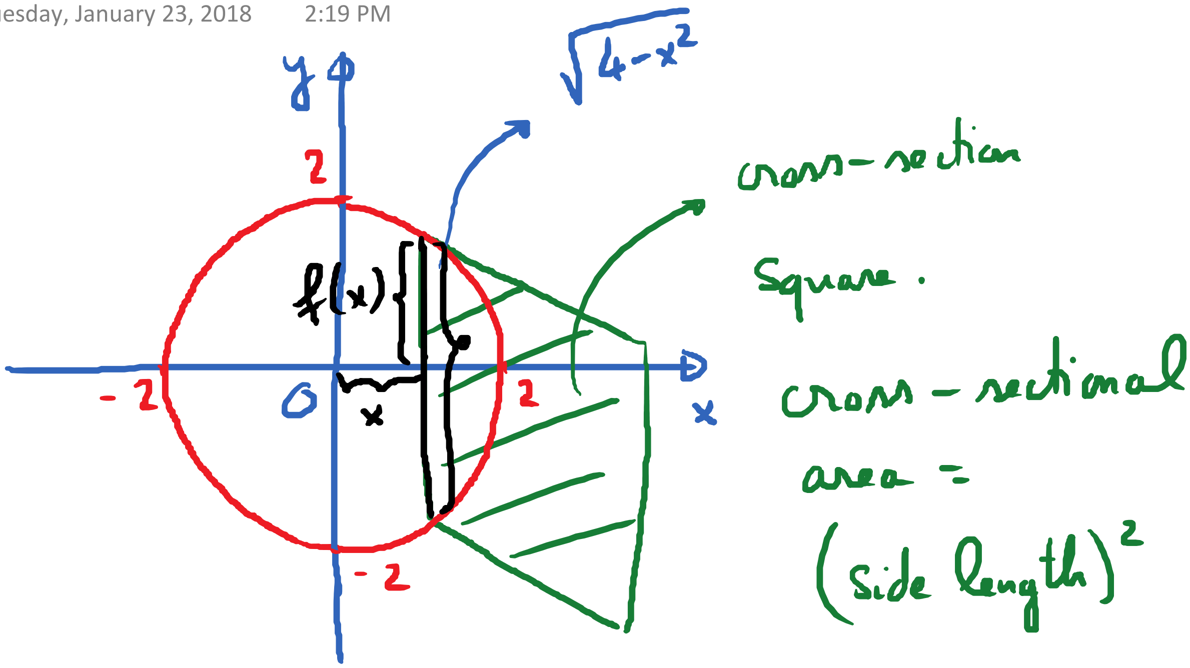
cross-sectional area at y = $(2\sqrt{1-y})^2$

$$\text{Volume of object} = \int_0^1 (2\sqrt{1-y})^2\,dy$$

$$= \int_0^1 4(1-y)\,dy = 4\int_0^1 (1-y)\,dy$$

$$= 4\cdot\left(y - \frac{y^2}{2}\right)\Big|_0^1 = 4\cdot\frac{1}{2} = \boxed{2}.$$

9.



Equation of circle: $x^2+y^2=4$

$$y=\pm\sqrt{4-x^2}$$

Side length $= 2\sqrt{4-x^2}$

cross-sectional area $= \left(2\sqrt{4-x^2}\right)^2$

$$V = 2\cdot\int_0^2 \left(2\sqrt{4-x^2}\right)^2 dx$$

$$= 2\cdot\int_0^2 4(4-x^2)\,dx = 8\cdot\int_0^2 (4-x^2)\,dx$$

$$= 8\cdot\left(4x-\frac{x^3}{3}\right)\Big|_0^2 = \boxed{\frac{108}{3}}$$

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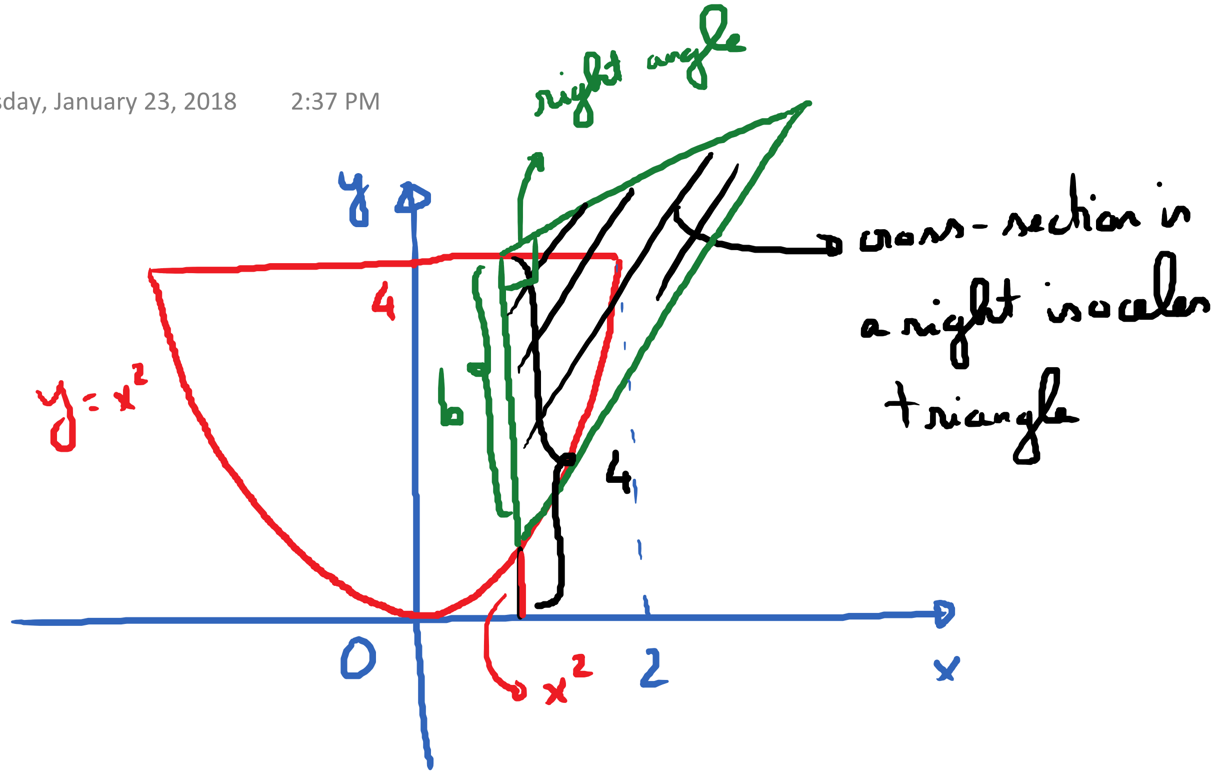
#10



Cross sectional area $= \frac{1}{2} b h$. But $b = h$ (Right isoceles) $\Big\} = \frac{1}{2} b^2$

$b = 4 - x^2$.

Cross sectional area $= \frac{1}{2} \cdot (4 - x^2)^2$

$$\text{Volume} = 2 \cdot \int_0^2 \frac{1}{2}(4 - x^2)^2 \, dx$$

$$= \int_0^2 (16 - 8x^2 + x^4) \, dx$$

$$= \left(16x - 8\frac{x^3}{3} + \frac{x^5}{5}\right)\Big|_0^2 = \boxed{\frac{256}{15}}$$

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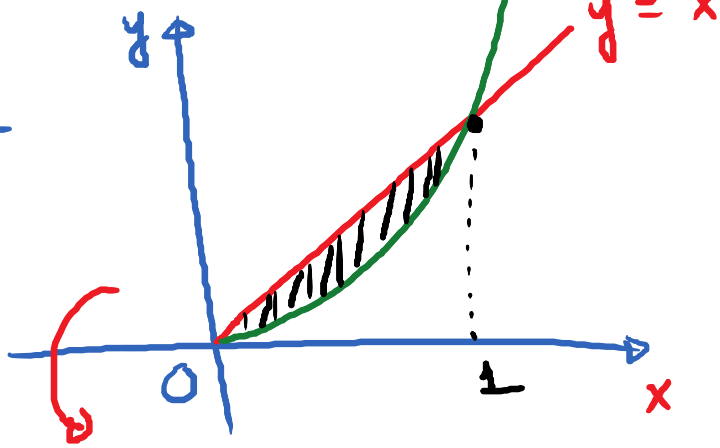
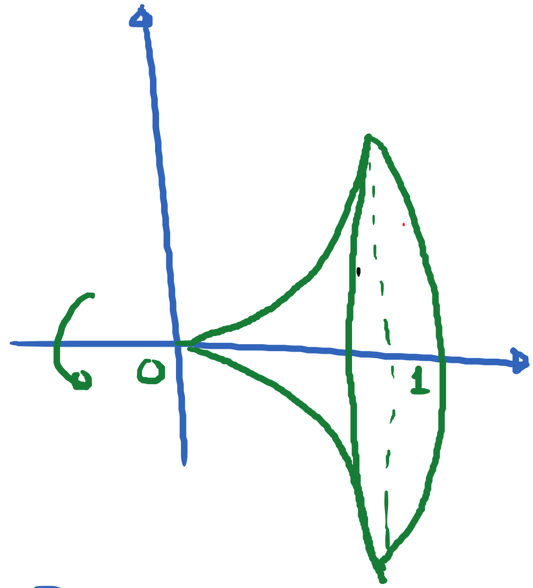
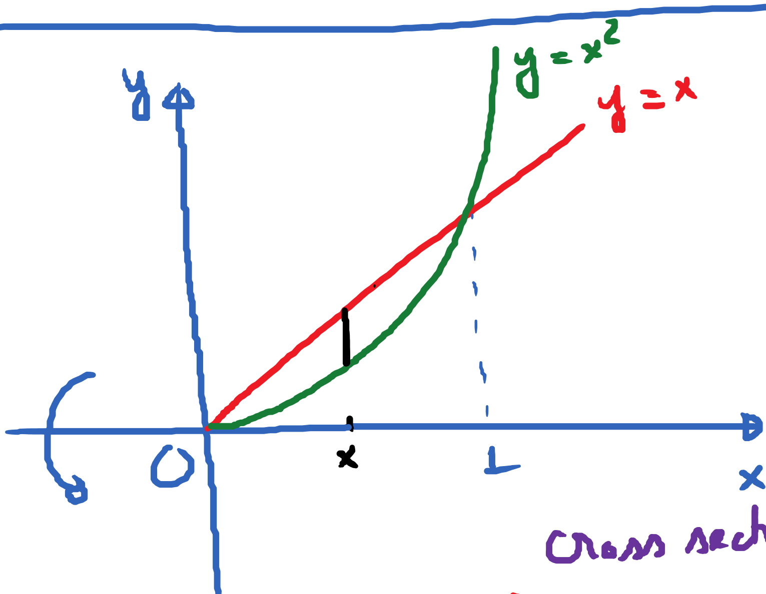
Washer Method:

E.g.

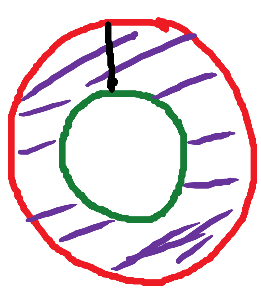




$$\pi \int_0^1 x^2 \, dx - \pi \int_0^1 x^4 \, dx = \cdots$$



Cross section of object is a washer.



Area of washer = area of outer circle − area of inner circle

$= \pi \cdot (\text{outer radius})^2 - \pi (\text{inner radius})^2$

$= \pi \cdot x^2 - \pi \cdot x^4$

cross-sectional area $= \pi \cdot x^2 - \pi \cdot x^4$.

$$V = \int_0^1 (\pi \cdot x^2 - \pi \cdot x^4)\, dx = \pi \int_0^1 (x^2 - x^4)\, dx$$

$= \cdots$