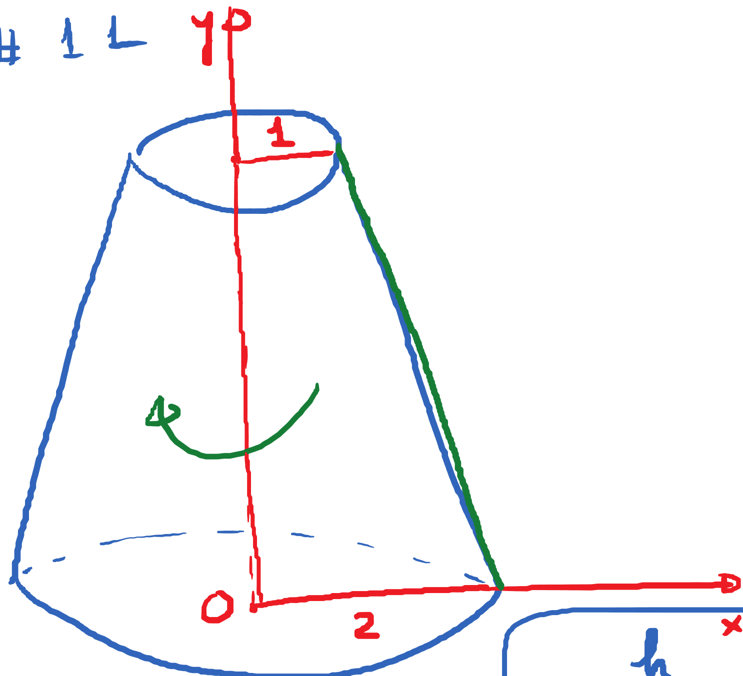
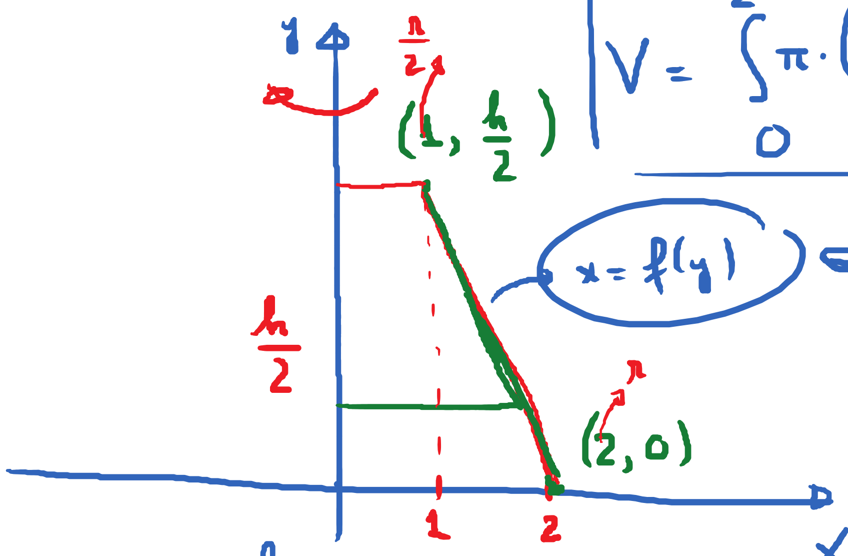
Thursday, January 25, 2018 1:12 PM

2.2 - #1L



$$\boxed{V = \int_0^{\frac{h}{2}} \pi \cdot \left(f(y)\right)^2 \, dy}$$



$$\text{Slope} = \frac{\frac{h}{2}}{-1} = -\frac{h}{2}.$$

$$2y = -hx + 2h$$

Point-Slope equation:

$$y - 0 = -\frac{h}{2}(x - 2)$$

$$y = -\frac{h}{2}x + h.$$

$$2y - 2h = -hx$$

$$x = \frac{2y - 2h}{-h}.$$

$$x = \boxed{-\frac{2}{h}y + 2}$$

2.3. Volumes by Cylindrical Shell.

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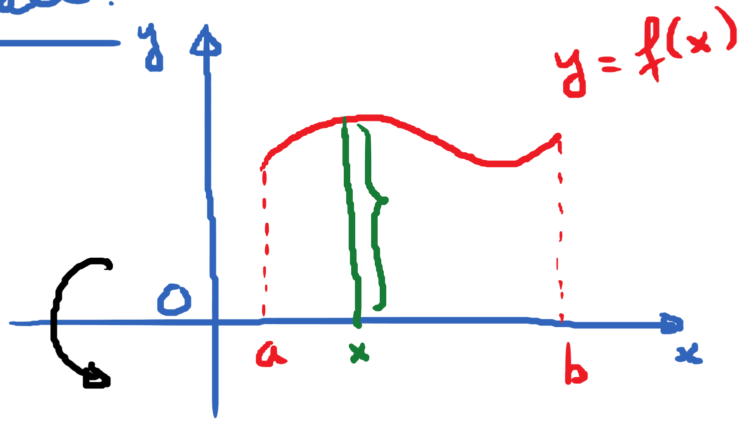
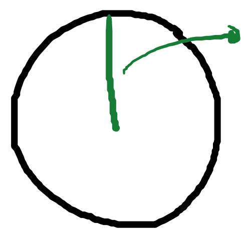
Recall: Disk Method

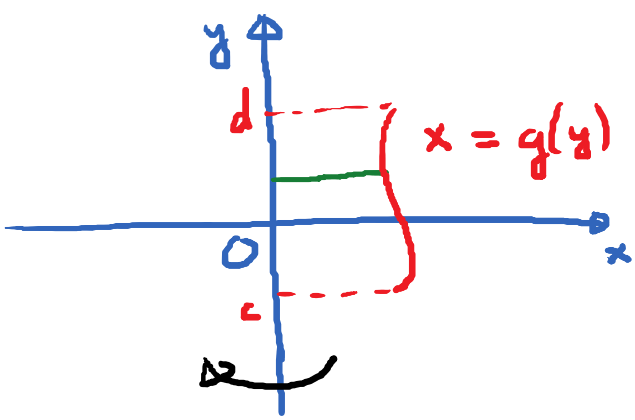


Cross section.

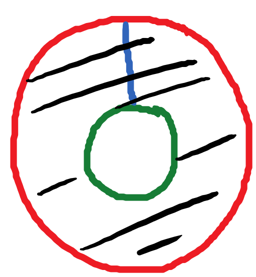


$$\text{Volume of solid obtained} = \int_a^b \pi\,(\text{radius})^2\,dx$$

$$= \int_a^b \pi\,(f(x))^2\,dx$$



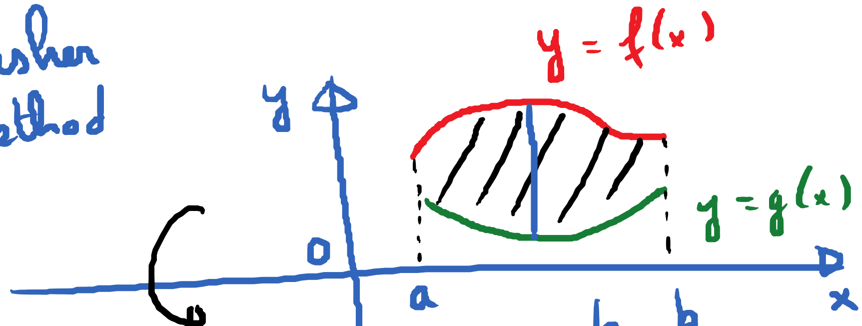
$$V = \int_a^b \pi\,(g(y))^2\,dy$$



cross sectional area

$$\pi(\text{outer radius})^2 - \pi(\text{inner radius})^2$$

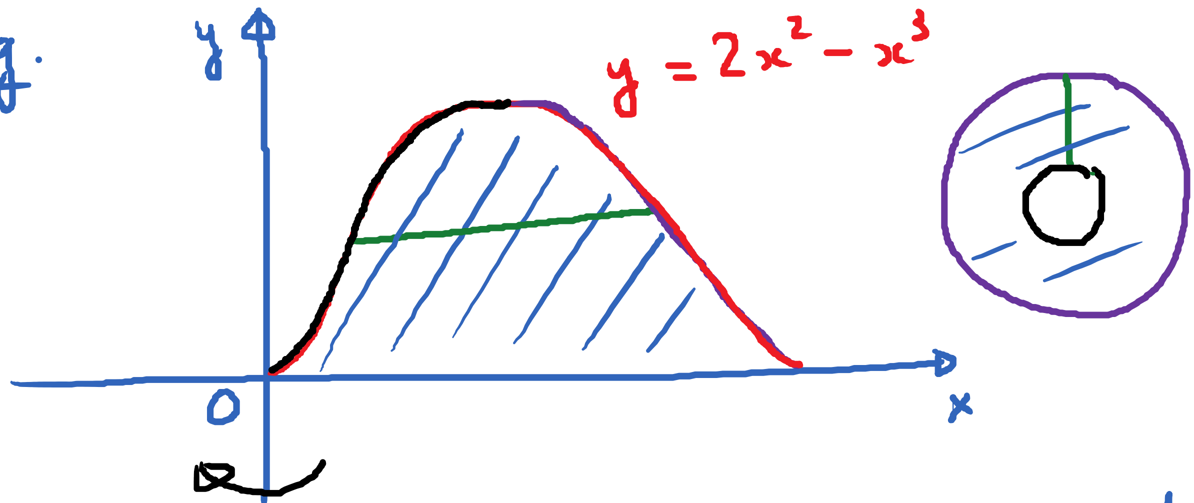
Washer Method



$$V = \int_a^b \left(\pi\,(f(x))^2 - \pi(g(x))^2\right) dx$$

Note: For disk/washer method, the cross-sections are perpendicular to the axis of revolution.

E.g.



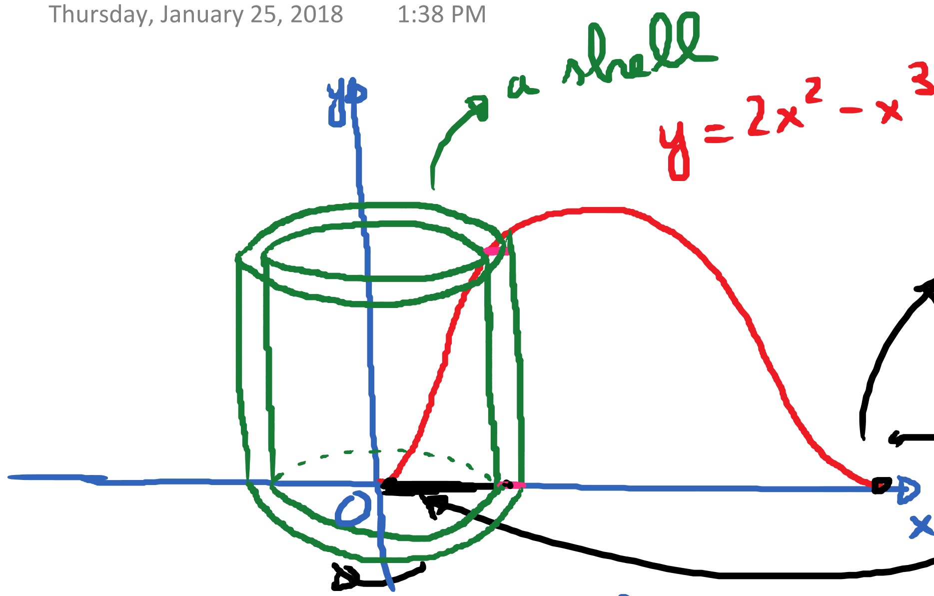
Rotate the region bounded by $y = 2x^2 - x^3$ and x-axis about the y-axis. $\longrightarrow$ obtain solid.

Q: Find the volume of solid.

Cross-sectional area = $\pi \cdot (\text{outer radius})^2 - \pi \cdot (\text{inner radius})^2$

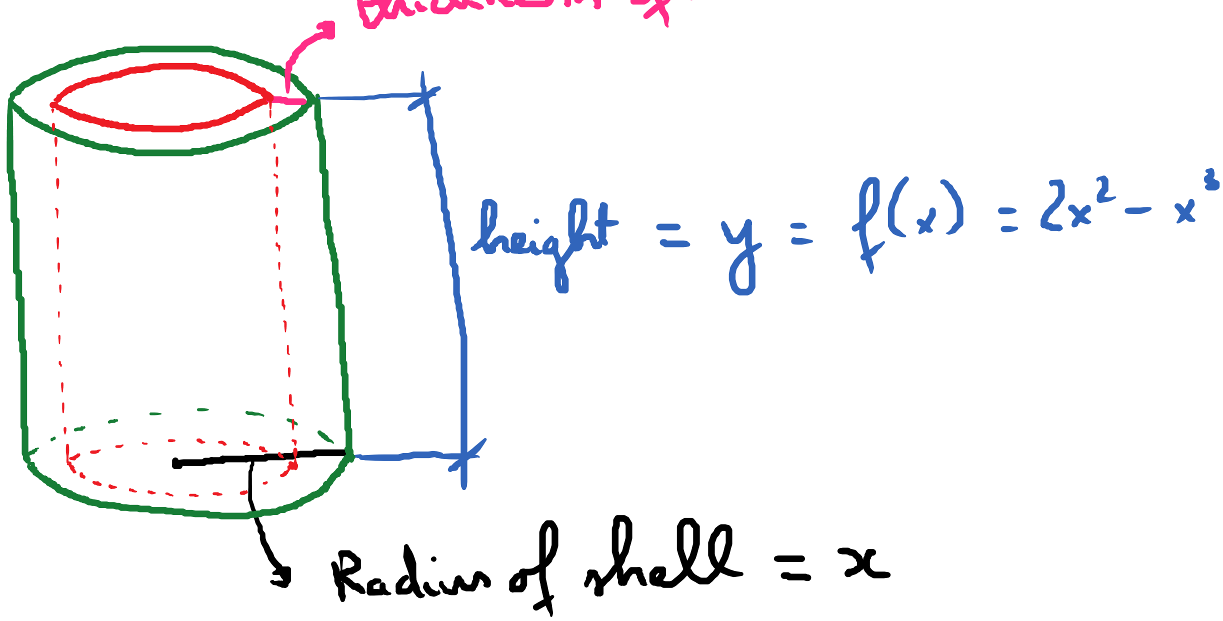
$\longrightarrow$ this is very hard to do b/c formulas for outer and inner radius are very complicated.

$\longrightarrow$ Shell method will make things easier.



$$2x^2 - x^3 = 0$$
$$x^2(2-x) = 0$$
$$x = 0 \; ; \; x = 2$$

Q: Find the volume of a generic shell



Volume of a generic shell at x

$$= 2\pi \cdot (\text{radius}) \cdot (\text{height}) \cdot (\text{thickness})$$
$$= 2\pi \cdot x \cdot (2x^2 - x^3) \cdot dx$$

Volume of object = Sum of volumes of shells

$$= \int_0^2 2\pi \cdot x \cdot (2x^2 - x^3) \cdot dx$$

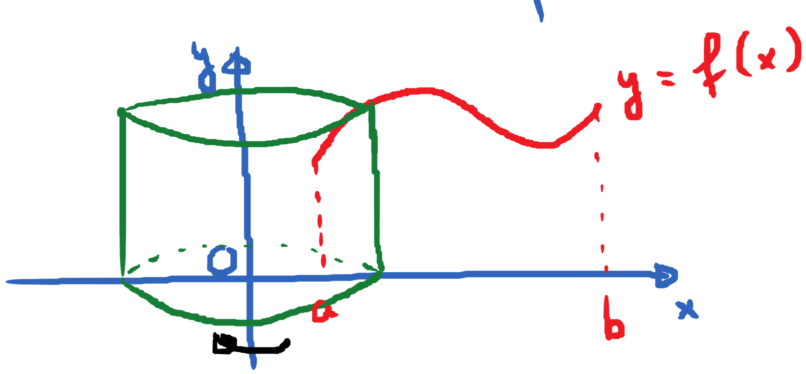
$$= 2\pi \cdot \int_0^2 (2x^3 - x^4)\, dx$$

$$= 2\pi \cdot \left(2 \cdot \frac{x^4}{4} - \frac{x^5}{5}\right)\Big|_0^2$$

$$= 2\pi \cdot \left(2 \cdot \frac{16}{4} - \frac{32}{5}\right)$$

$$= 2\pi \cdot \left(8 - \frac{32}{5}\right) = \boxed{\frac{16\pi}{5}}$$

General Formula for Shell Method



$$\boxed{V = \int_a^b 2\pi \cdot x \cdot f(x) \cdot dx}$$