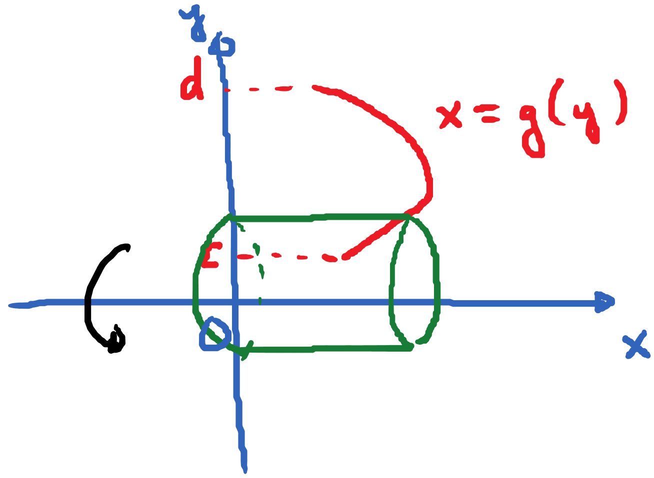
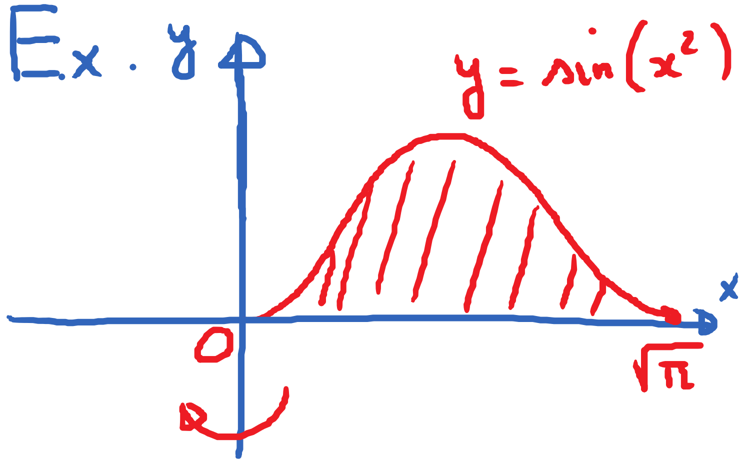
Thursday, January 25, 2018 1:55 PM

$$V = \int_c^d 2\pi y \cdot g(y) \cdot dy.$$



Ex.



Rotate region bounded by

$y = \sin(x^2)$; $0 \leq x \leq \sqrt{\pi}$

and x-axis.

about the y-axis.

Find the volume of the object obtained.

$$V = \int_0^{\sqrt{\pi}} 2\pi \cdot x \cdot \sin(x^2)\, dx = \boxed{2}\pi \int_0^{\sqrt{\pi}} \boxed{x} \cdot \sin(x^2)\boxed{dx}$$

$\sqrt{\pi} \to x = \sqrt{\pi} \to u = (\sqrt{\pi})^2 = \pi$

$0 \to \boxed{x = 0} \to u = 0^2 = 0$

Let $u = x^2$. $du = 2x\,dx$

$$V = \pi \int_{\boxed{0}}^{\boxed{\pi}} \sin(u)\, du = \pi \cdot (-\cos(u)) \Big|_0^{\pi}$$

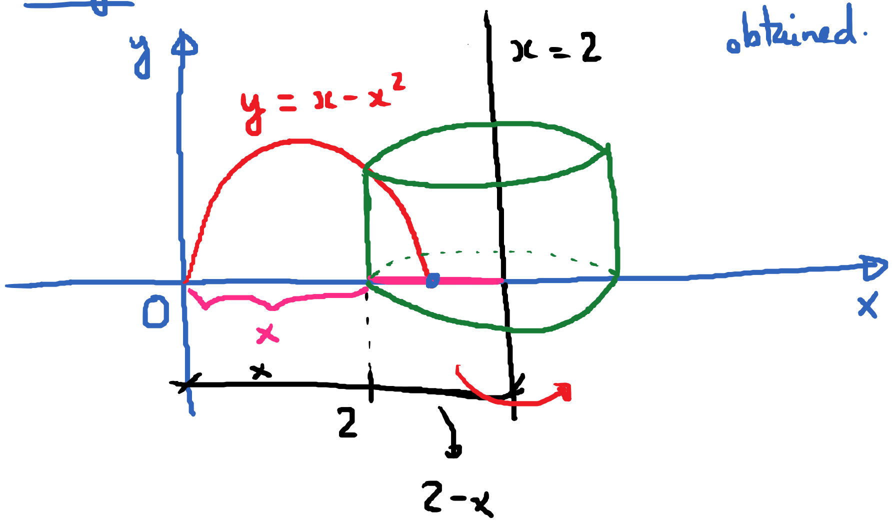
$$= \pi \cdot (1 + 1) = \boxed{2\pi}.$$

$\boxed{0}$ $= \pi \cdot (1 + 1) = \boxed{2\pi \cdot}$

* Axis of rotation is any vertical line or any horizontal line.

E.g.

Find the volume of solid obtained.

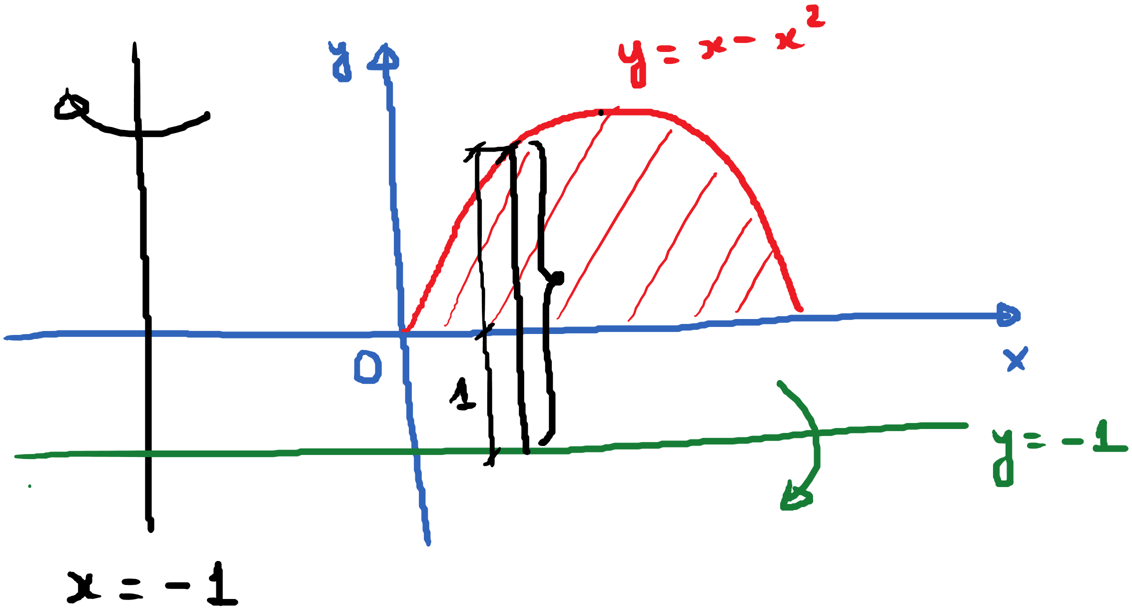


Radius of shell $= 2 - x$.

Height of shell $= x - x^2$

$$V = \int_0^1 2\pi \cdot (2-x) \cdot (x - x^2)\, dx$$

$= \ldots\ldots$



① Rotate about $x = -1$. Find Volume.

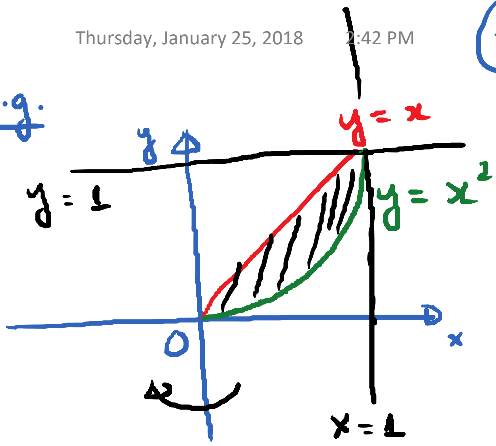
② Rotate about $y = -1$. Find volume.

① $V = 2\pi \int_0^1 (1 + x)(x - x^2)\,dx$

② $V = \int_0^1 \pi \cdot (1 + x - x^2)^2\,dx$

Thursday, January 25, 2018 2:42 PM

E.g.



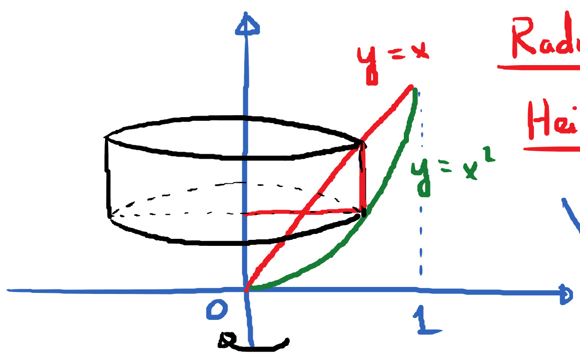
(1) Set up the integral to find the volume obtained by rotating the shaded region about the y-axis using the shell method

(2) Using disk/washer method

(3) Rotate about $x=1$. Using shell method.

(4) Rotate about $y=1$. Using disk/washer method.

(1)



Radius: x

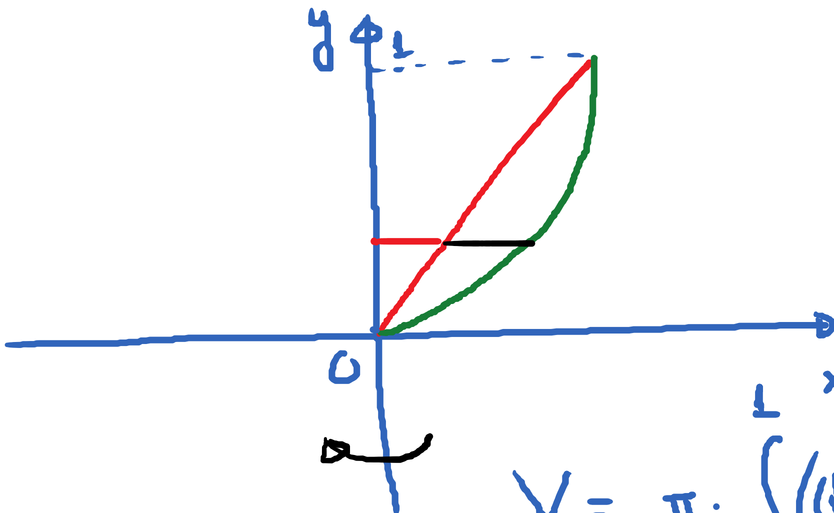
Height: $x - x^2$

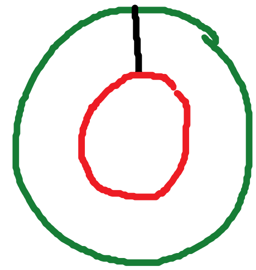
$$V = 2\pi \int_0^1 x \cdot (x - x^2) \cdot dx$$

(2)

Inner radius = y

Outer radius = $\sqrt{y}$



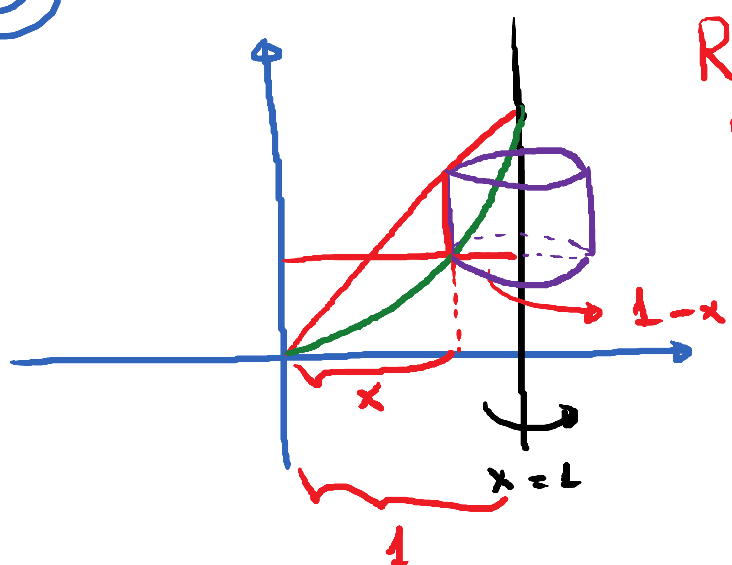


$$V = \pi \cdot \int_0^1 \left((\sqrt{y})^2 - y^2 \right) dy .$$

(3)

Radius: $1-x$

Height: $x - x^2$

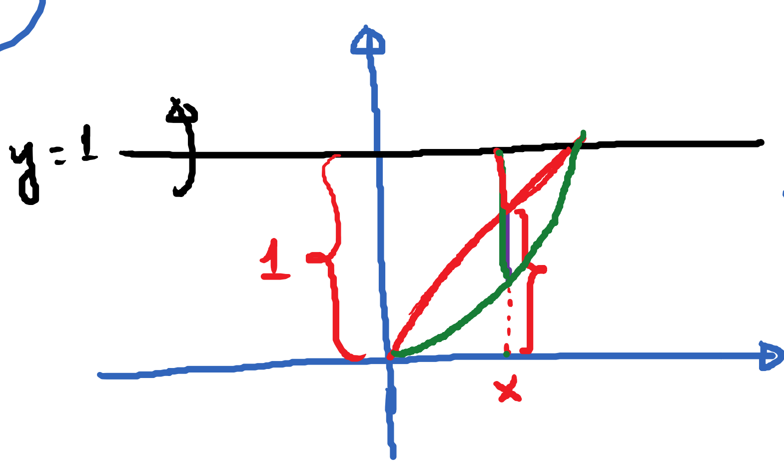


$$V = 2\pi \cdot \int_0^1 (1-x)(x-x^2)\,dx$$

(4)

Inner Radius: $1-x$

Outer Radius: $1-x^2$



$$V = \pi \int_0^1 \left((1-x^2)^2 - (1-x)^2 \right) dx$$