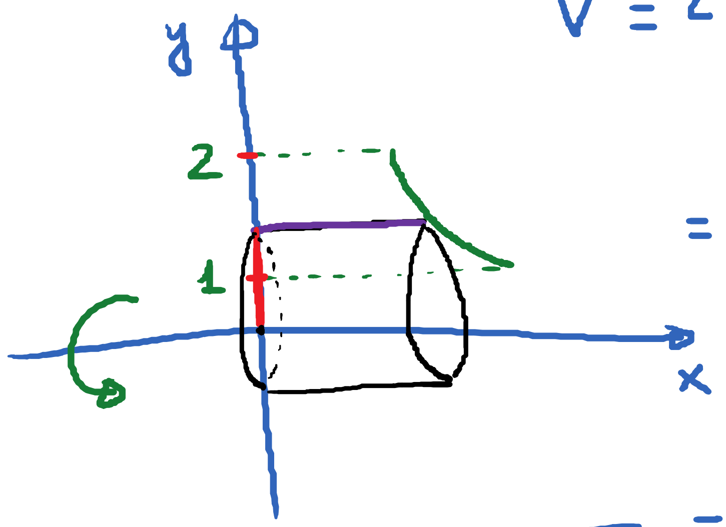
HW3 # 10

$x = \frac{1}{1+y^2}$; $y = 1$ and $y = 2$

$$V = 2\pi \int_1^2 (\text{radius})(\text{height})\, dy$$



$$= 2\pi \int_1^2 y \cdot \frac{1}{1+y^2}\, dy$$

du

$$= \boxed{2}\pi \int_1^2 \frac{y}{1+y^2}\, dy$$

Let $u = 1 + y^2$. $du = 2y\, dy$

$$\pi \int_2^5 \frac{du}{u} = \pi \cdot \ln|u| \Big|_2^5 = \pi \cdot (\ln(5) - \ln(2))$$

$$= \pi \cdot \ln\left(\frac{5}{2}\right)$$

2.4. Arc Lengths and Surface Areas

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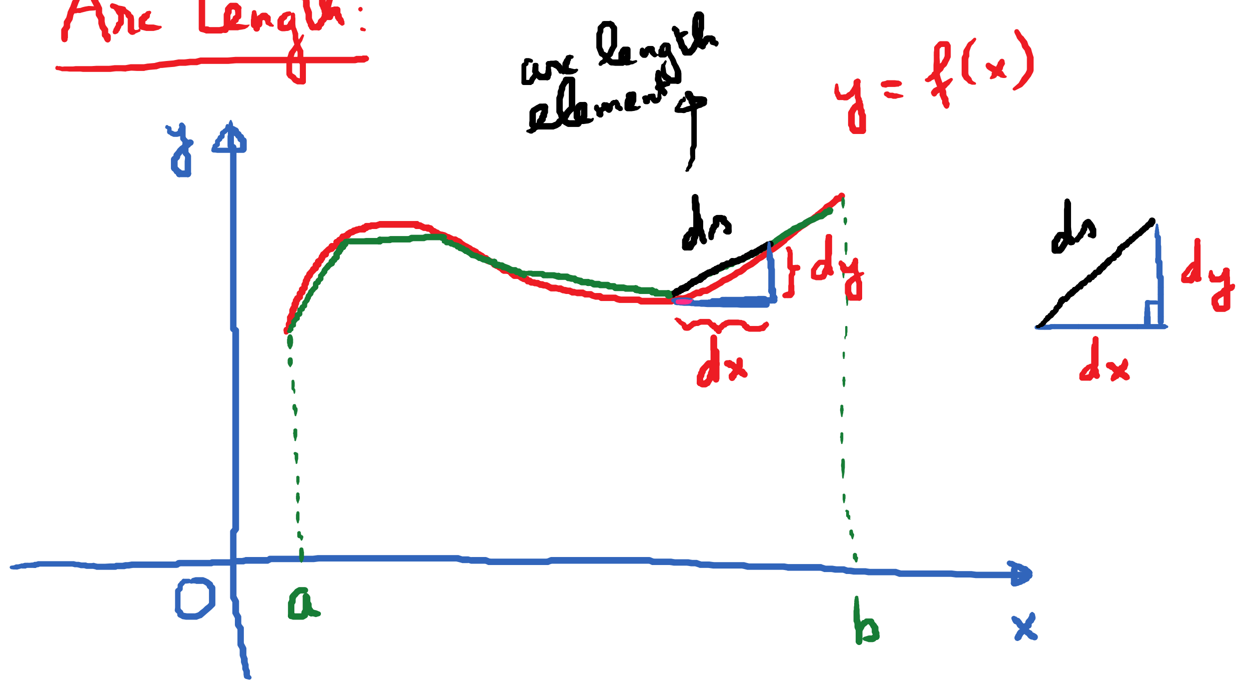
Arc Length:



Find length of the curve $y = f(x)$; $a \leq x \leq b$

$$L = \boxed{\int_a^b ds} = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

Where the formula comes from:

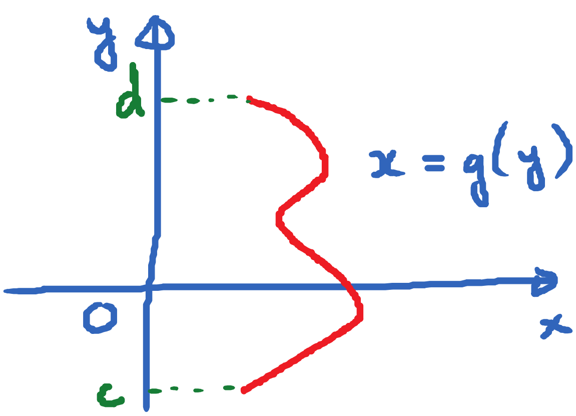
$$(ds)^2 = (dx)^2 + (dy)^2 \quad (\text{Pythagorean Theorem})$$

$$ds = \sqrt{(dx)^2 + (dy)^2}$$

$$ds = \sqrt{(dx)^2\left[1 + \frac{(dy)^2}{(dx)^2}\right]} = \sqrt{1 + \left(\boxed{\frac{dy}{dx}}\right)^2}\, dx$$

$$ds = \sqrt{1 + [f'(x)]^2}\, dx$$

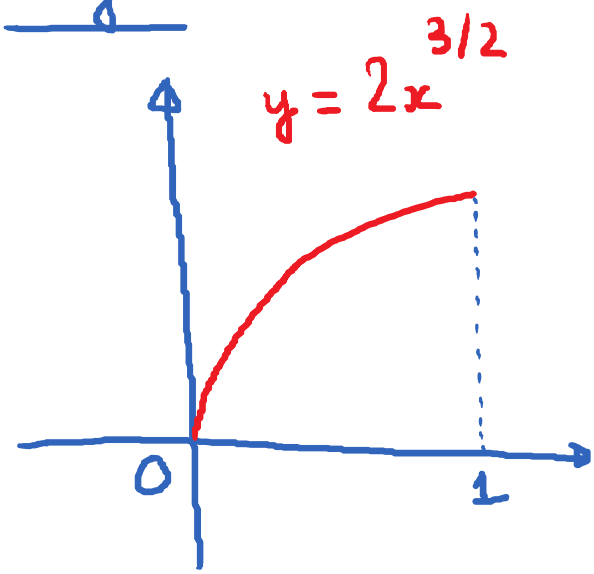
$$L = \int_a^b ds = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$



Length of curve $x = g(y)$; $c \le y \le d$

$$L = \int_c^d ds = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy.$$

E.g. 1:



Find length of the curve

$$y = 2x^{3/2}, \quad 0 \le x \le 1.$$

$$\frac{dy}{dx} = 3\sqrt{x}$$

$$L = \int_0^1 \sqrt{1 + (3\sqrt{x})^2}\, dx$$

$$L = \frac{1}{9}\int_0^1 \sqrt{1 + 9x}\, \boxed{9dx}$$

Let $u = 1 + 9x$. $du = 9dx$

$$L = \frac{1}{9}\int_1^{10} \sqrt{u}\, du = \frac{1}{9}\int_1^{10} u^{\frac{1}{2}}\, du = \frac{1}{9}\cdot\frac{2u^{\frac{3}{2}}}{3}\Bigg|_1^{10}$$

$$= \frac{2}{27}\left((10)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right)$$

$$= \frac{2}{27}\left(\sqrt{1000} - 1\right) = \boxed{\frac{2}{27}\left(10\sqrt{10} - 1\right)}$$

HW4 #1

$$\frac{dy}{dx} = \frac{x}{2} - \frac{1}{2x}$$

$$L = \int_1^{3e} \sqrt{1 + \left(\frac{x}{2} - \frac{1}{2x}\right)^2}\, dx$$

$$= \int_1^{3e} \sqrt{1 + \frac{x^2}{4} - \frac{1}{2} + \frac{1}{4x^2}}\, dx = \int_1^{3e} \sqrt{\frac{x^2}{4} + \frac{1}{2} + \frac{1}{4x^2}}\, dx$$

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$$= \int_1^{3e} \sqrt{\left(\frac{x}{2}\right)^2 + 2\cdot\frac{x}{2}\cdot\frac{1}{2x} + \left(\frac{1}{2x}\right)^2}\, dx$$

$$= \int_1^{3e} \sqrt{\left(\frac{x}{2} + \frac{1}{2x}\right)^2}\, dx = \int_1^{3e} \left(\frac{x}{2} + \frac{1}{2x}\right) dx$$

$$= \left(\frac{x^2}{4} + \frac{1}{2}\cdot \ln|x|\right)\Big|_1^{3e}$$

$$= \left[\frac{(3e)^2}{4} + \frac{1}{2}\cdot\ln(3e)\right] - \left[\frac{1}{4} + \frac{1}{2}\ln(1)\right]$$

$$= \left[\frac{9e^2}{4} + \frac{1}{2}\ln(3) + \frac{1}{2}\right] - \left[\frac{1}{4}\right]$$

$$= \boxed{\frac{9e^2}{4} + \frac{1}{2}\ln 3 + \frac{1}{4}}.$$