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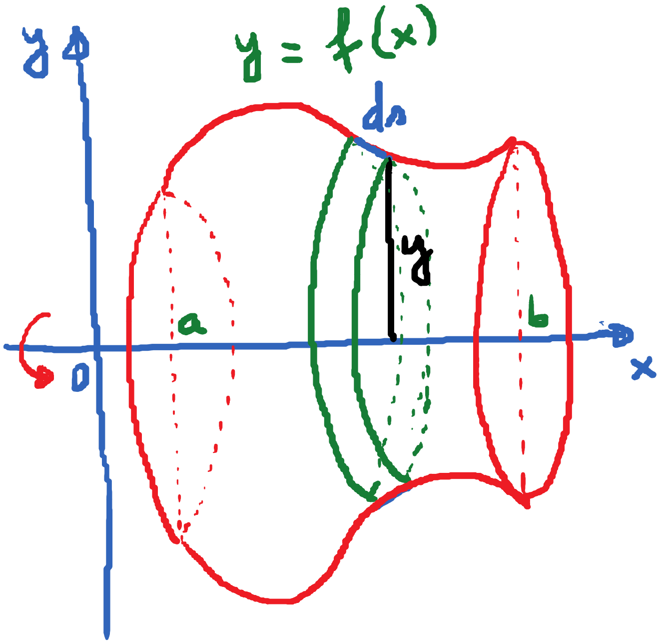
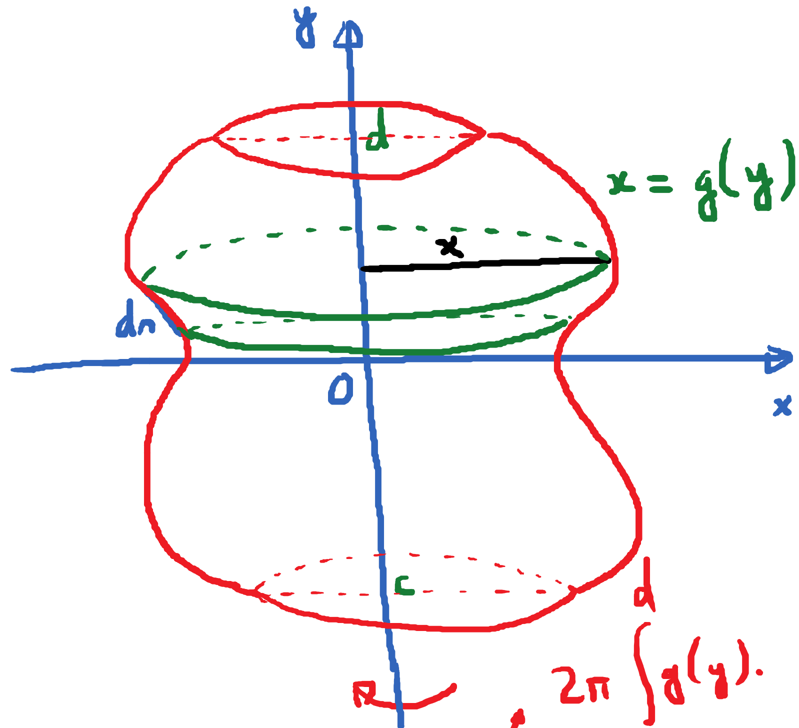
Surface Area of Surface of Revolution.





Surface area S

$$\boxed{S = 2\pi \int_a^b y\,ds}$$

$$S = 2\pi \int_c^d x\,ds$$

$$2\pi \int_c^d g(y)\sqrt{1+[g'(y)]^2}\,dy$$

$$\underbrace{2\pi y\,ds}_{\text{area of small patch}}$$

$$2\pi \int_a^b f(x)\sqrt{1+[f'(x)]^2}\,dx$$

Make $S = 2\pi \int_a^b y \boxed{ds}$ workable.

$$= \boxed{2\pi \int_a^b f(x) \cdot \sqrt{1 + [f'(x)]^2}\, dx} \quad (*)$$

Replace ds by $\sqrt{1 + [f'(x)]^2}\, dx$

Replace y by $f(x)$

$$ds = \sqrt{(dx)^2 + (dy)^2}$$

$$= \sqrt{(dy)^2 \left[\frac{(dx)^2}{(dy)^2} + 1 \right]}$$

$$= \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

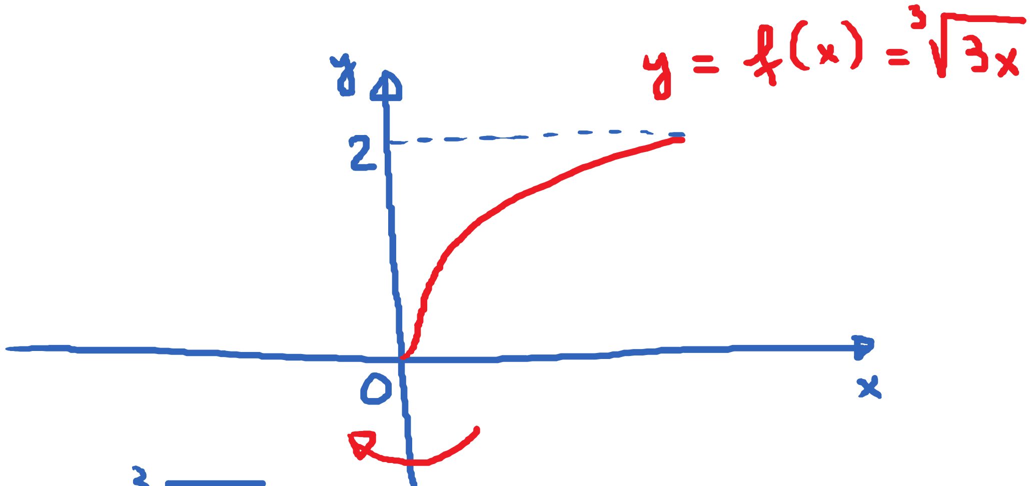
Note: If $(*)$ is too hard to evaluate, we first write x as a function of y; say, $x = g(y)$

$$S = 2\pi \int_a^b y\,ds = 2\pi \int_a^b y\sqrt{1 + [g'(y)]^2}\,dy.$$

Second formula:

$$S = 2\pi \int_c^d x\,ds = 2\pi \int_c^d g(y)\sqrt{1 + [g'(y)]^2}\,dy$$

E.g.



Rotate $y = \sqrt[3]{3x}$ about y-axis ; $0 \le y \le 2$

Find the surface area of the object obtained.

$$S = 2\pi \int_0^2 x\,ds$$

$$ds = \sqrt{1 + [f'(x)]^2}\, dx$$

$$f(x) = \sqrt[3]{3x} \longrightarrow f'(x) \stackrel{?}{=} \frac{(3x)^{-2/3}}{3}$$

$$ds = \sqrt{1 + \left[\frac{(3x)^{-2/3}}{3}\right]^2}\, dx$$

$$S = 2\pi \int_0^2 x \cdot \sqrt{1 + \frac{(3x)^{-4/3}}{9}}\, dx$$

$\longrightarrow$ pretty hard!

$$ds = \sqrt{1 + [g'(y)]^2}\, dy$$

How to get formula for $g(y)$?

$$y = \sqrt[3]{3x} \longrightarrow y^3 = 3x \longrightarrow x = \boxed{\frac{y^3}{3}}$$

$$g'(y) = y^2.$$

$$ds = \sqrt{1 + (y^2)^2}\, dy = \sqrt{1 + y^4}\, dy$$

$$S = 2\pi \int_0^2 x\, ds = 2\pi \int_0^2 x \cdot \sqrt{1 + y^4}\, dy$$

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$$S = 2\pi \int_0^2 \frac{y^3}{3}\sqrt{1+y^4}\,dy$$

$$S = \frac{1}{4}\cdot\frac{2\pi}{3}\int_0^2 4y^3\sqrt{1+y^4}\,dy.$$

Let $u = 1 + y^4$. $du = 4y^3\,dy$

$$S = \frac{\pi}{6}\int_1^{17}\sqrt{u}\,du = \frac{\pi}{6}\int_1^{17} u^{1/2}\,du$$

$$= \frac{\pi}{6}\cdot\frac{2u^{3/2}}{3}\Bigg|_1^{17} = \frac{\pi}{9}\cdot u^{3/2}\Bigg|_1^{17}$$

$$= \frac{\pi}{9}\left((17)^{3/2} - 1\right) = \frac{\pi}{9}\left(17\sqrt{17} - 1\right)$$

HW 4 #9: $S = 2\pi\int_1^4 y\,ds$. $y = \sqrt{x}$; $f'(x) = \frac{1}{2\sqrt{x}}$

$$ds = \sqrt{1+[f'(x)]^2}\,dx = \sqrt{1+\left(\frac{1}{2\sqrt{x}}\right)^2}\,dx$$

$$ds = \sqrt{1 + \frac{1}{4x}}\, dx$$

$$S = 2\pi \int_1^4 \sqrt{x} \cdot \sqrt{1 + \frac{1}{4x}}\, dx$$

$$= 2\pi \int_1^4 \sqrt{x \cdot \left(1 + \frac{1}{4x}\right)}\, dx$$

$$= 2\pi \cdot \int_1^4 \sqrt{x + \frac{1}{4}}\, dx$$ Let $u = x + \frac{1}{4}$.