2.5. Physical Applications

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1. Mass and Density
2. Work done by a force.
3. Hydrostatic force and pressure

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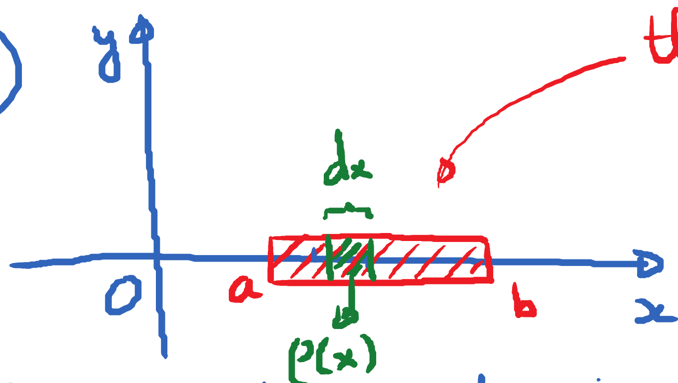


thin rod along x-axis.

$a \leq x \leq b$

Density at a point x is given by $\boxed{\rho(x)}$ density function (non-constant density)

Q: Find m_{rod} (weight of rod)

density = mass per unit length

Consider dx : small segment of the rod. Within this small segment, we can consider density to be constant and equal $\rho(x)$.

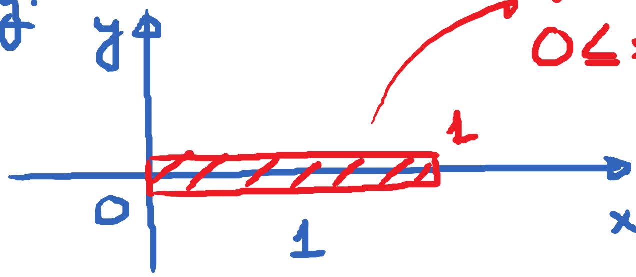
Weight of this small segment $= \rho(x)\,dx$

Weight of the entire rod $= \sum$ weights of these segments

Weight of ... segments

So, $\boxed{m_{rod} = \int_a^b \rho(x)\,dx}$

E.g.



thin rod. Density function:

$0 \le x \le 1$ $\rho(x) = 5(x+2)^{-2}$

Find m_{rod} ?

$$m_{rod} = \int_0^1 5(x+2)^{-2}\,dx = 5\int_0^1 (\underbrace{\boxed{x+2}}_{u})^{-2}\,\boxed{dx}$$

Let $u = x+2$. $du = dx$

$$= 5\int_2^3 u^{-2}\,du = 5\cdot\frac{u^{-1}}{-1}\Big|_2^3$$

$$\left(\int u^n\,du = \frac{u^{n+1}}{n+1} \quad (n \neq -1) \qquad \int u^{-1}\,du = \ln|u|\right)$$

$$= -\frac{5}{u}\Big|_2^3$$

$$= -\frac{5}{3} - \left(-\frac{5}{2}\right) = -\frac{5}{3} + \frac{5}{2} = \boxed{\frac{5}{6}}$$

* Circular object with radial density.

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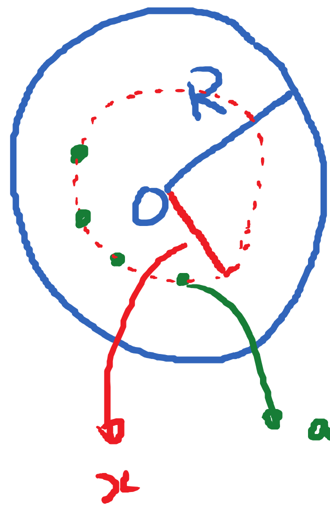


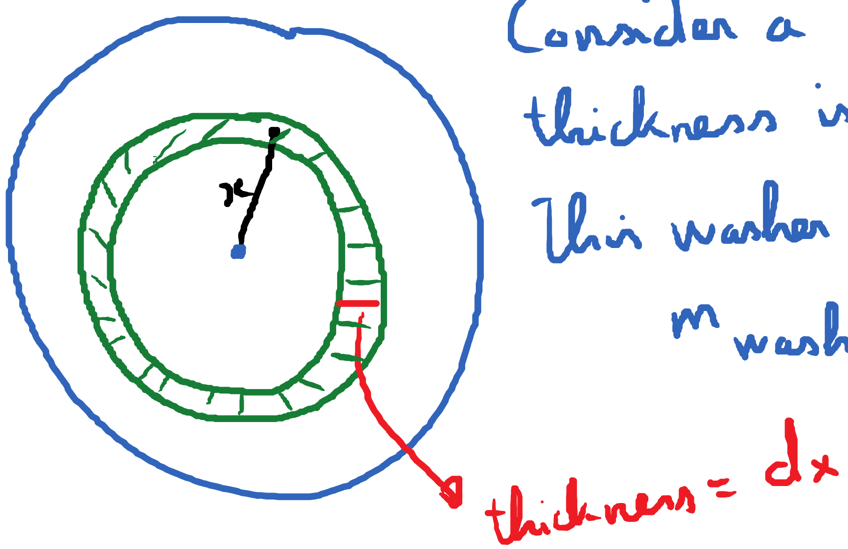
Disk of radius R.

Radial density is given by a function $\rho(x)$

at every point on the circle with radius x, density is $\rho(x)$

→ Q: $m_{disk} = ?$

Consider a "thin" washer whose thickness is dx



This washer has uniform density $\rho(x)$

$$m_{washer} = \underbrace{(\text{density})}_{\rho(x)} \cdot \underbrace{(\text{area})}$$

$$= \underbrace{(\text{circumference})}_{} \cdot (\text{thickness})$$

$$\boxed{2\pi x \cdot dx} \to \text{area}$$

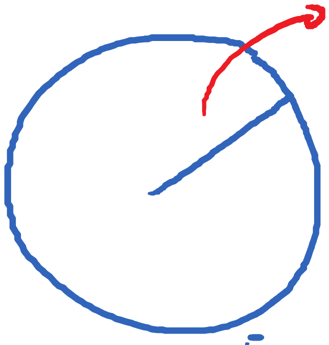
$$m_{washer} = \rho(x) \cdot 2\pi x \, dx$$

$$= 2\pi x \, \rho(x) \, dx$$

$$m_{disk} = \int_0^R 2\pi x \rho(x)\, dx$$

$$\boxed{m_{disk} = 2\pi \int_0^R x \rho(x)\, dx}$$

E.g.



$R = 5\,cm$.

Radial density $= \rho(x) = e^{-x^2}$ (g/cm^2)

$\longrightarrow$ Find m_{disk}?

$$m_{disk} = -\boxed{2}\pi \int_0^5 \boxed{x} e^{-x^2} (\boxed{-dx}).$$

let $u = -x^2$

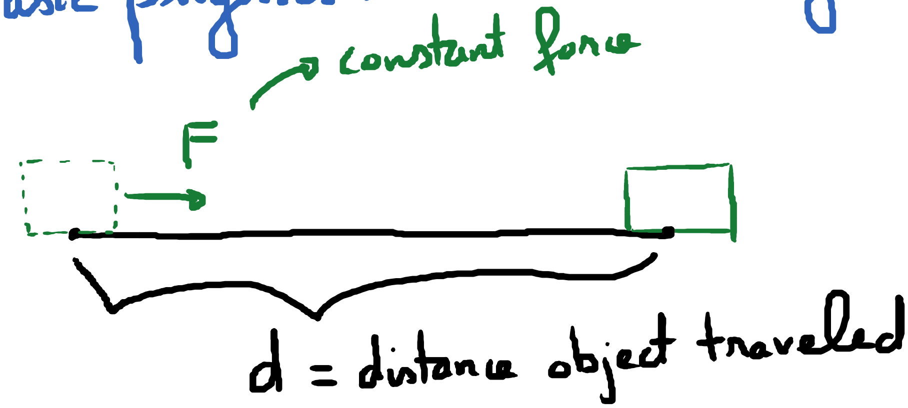
$du = -2x\, dx$

$$= -\pi \int_0^{-25} e^u\, du = -\pi \cdot e^u \Big|_0^{-25}$$

$$= \boxed{-\pi \cdot \left(e^{-25} - 1\right)}$$

2 Work done by a variable force

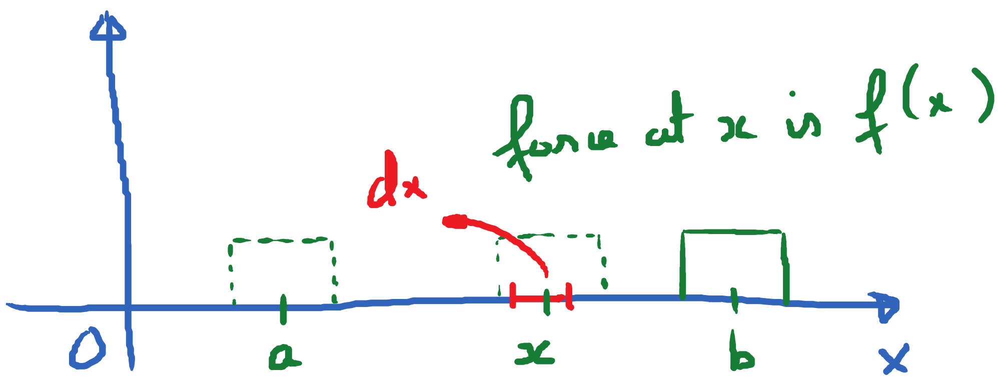
Basic physics: work done by constant force



Work done by F = W = (Force) · (distance)

$$\boxed{W = F \cdot d}$$

What if the force is changing?



Force is given by a function $f(x)$. (changes based on x)

$$W = ?$$

Divide the distance from a to b into really small distances dx.

Work done in moving the object a small distance dx is $\underbrace{f(x)}_{\text{force}} \cdot \underbrace{dx}_{\text{distance}}$ (we can consider the force to be constant throughout that really small distance)

Total work done in moving object from a to b

= $\sum$ work done in moving object in these small distances

$$= \int_a^b f(x)\,dx$$

$$\boxed{W = \int_a^b f(x)\,dx}$$

E.g. Find work done in moving object along the x-axis from $x = 1$ cm to $x = 2$ cm with the variable force $f(x) = \frac{12}{x^2}$ (N).

$$W = \int_1^2 \frac{12}{x^2}\,dx = 12 \cdot \int_1^2 x^{-2}\,dx$$

$$= 12 \cdot \frac{x^{-1}}{-1}\Big|_1^2 = 12 \cdot \left(-\frac{1}{x}\right)\Big|_1^2$$

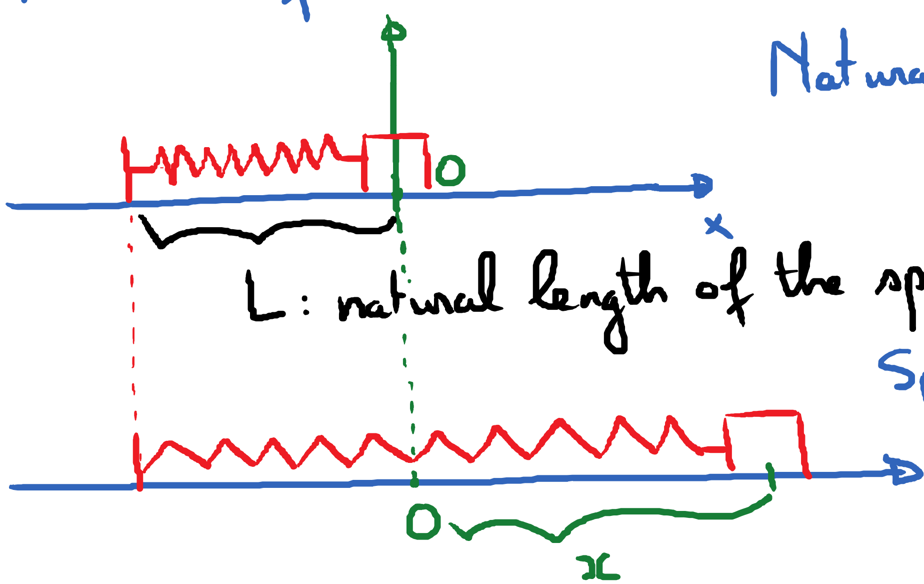
$$= 12 \cdot \left(-\frac{1}{2} + 1\right)$$

$$= 6 \text{ J}$$

* Work done in stretching a spring

Review of Hooke's Law in physics

Natural position of spring



Spring is stretched to the position x from O.