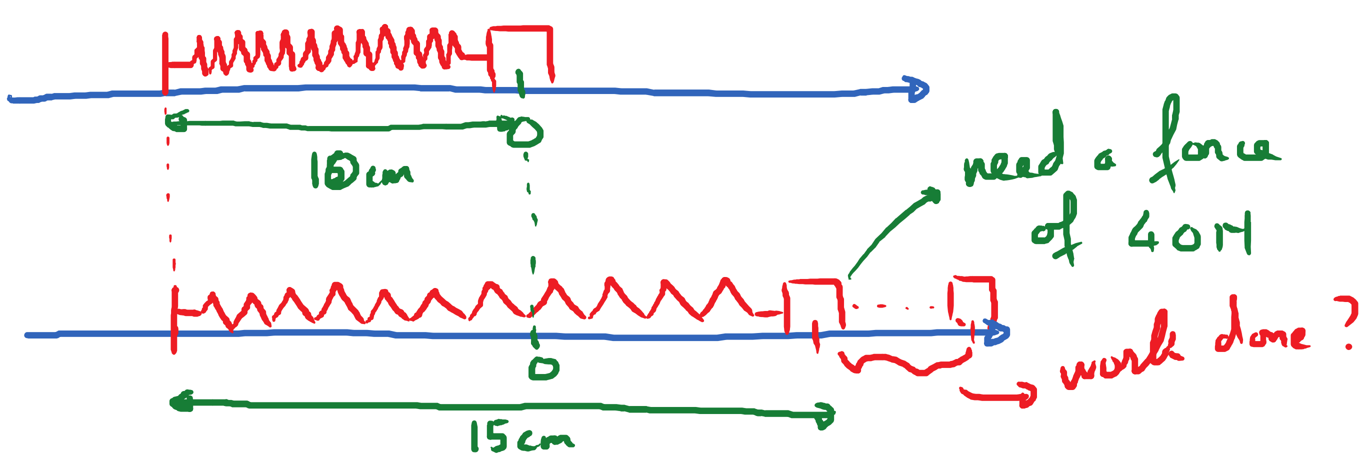
F: force required to maintain the spring in this stretched position. (distance x from 0)

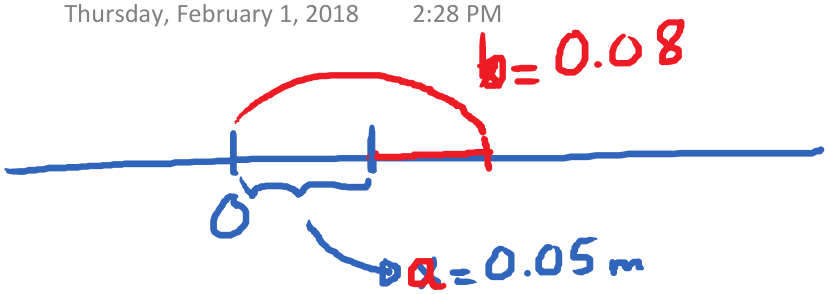
Hooke's law: $F(x) = k \cdot x$

(k is the spring constant)
(Hooke's constant)

E.g. Spring has natural length $L = 10$ cm.

* A force of 40 N is required to hold the spring when it is stretched from its natural length to a length of 15 cm.
* Find the work done in stretching the spring from 15 cm to 18 cm.



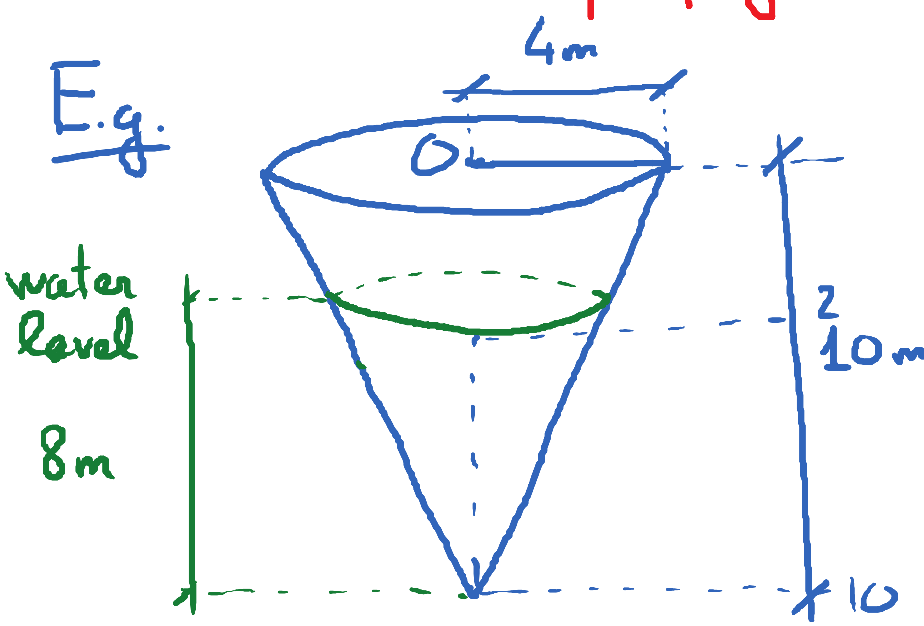


$$F = kx \longrightarrow 40 = k\cdot(0.05) \longrightarrow k = 800$$

$$W = \int_{0.05}^{0.08} 800x\,dx = 800\cdot \frac{x^2}{2}\Bigg|_{0.05}^{0.08} = \cdots$$

* Work done in pumping water (liquid) out of a tank

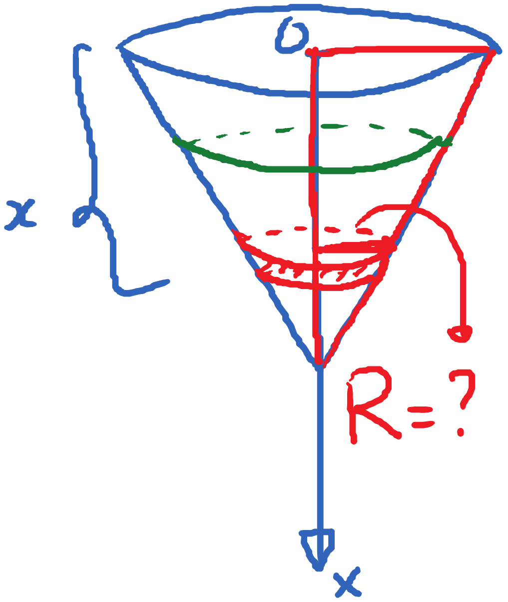
E.g.



Tank: shape is an inverted cone as shown.

Density of water is $1000\ kg/m^3$

Find the work done to empty tank by pumping water over the top of the tank.



Strategy:

* Find the work done in pumping a very "thin" slice of water out of the tank.
* Integrate that along the entire body of water to find the total work done.

$$\text{Work} = (\text{force})\cdot(\text{distance})$$

$$= (\text{gravity on slice})\cdot(\text{distance})$$

$$= \left(\underbrace{m_{slice}}\cdot g\right)\cdot(\text{distance})$$

$$= \left(\underbrace{(\text{volume}_{slice})(\text{density})}_{\text{mass}}\cdot g\right)\cdot(\text{distance})$$

work done in moving 1 slice

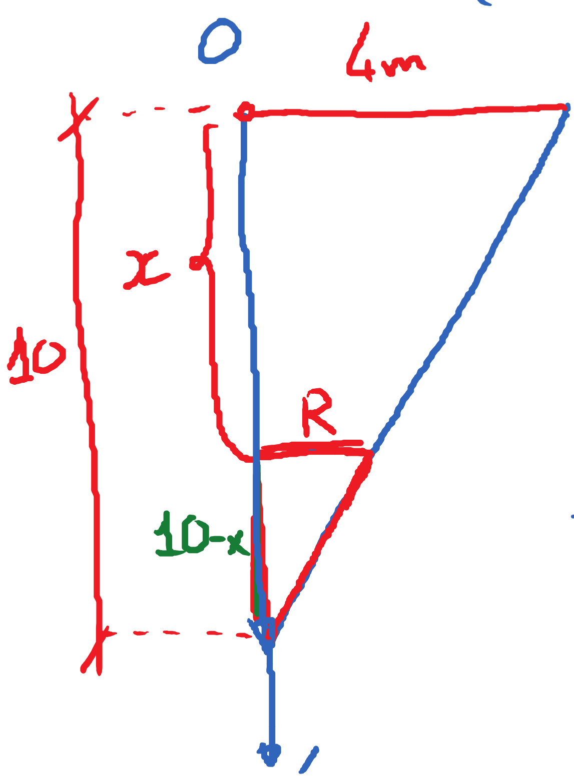
$$= (\text{density})\cdot(\text{volume}_{slice})\cdot g\cdot\overbrace{(\text{distance})}^{x}$$

→ It comes down to find the formula for the volume of a small slice.

Thursday, February 1, 2018 2:45 PM

$$V_{slice} = (\text{base area}) \cdot \underbrace{(\text{thickness})}_{dx}$$

$$= \pi \cdot (\text{radius})^2 \cdot dx$$



Find R in terms of x.

$$R = \frac{20-2x}{5}$$

$$\frac{R}{4} = \frac{10-x}{10}$$

$$R = \frac{10-x}{10} \cdot 4 = \frac{20-2x}{5}$$

$$V_{slice} = \pi \cdot \left(\frac{20-2x}{5}\right)^2 \cdot dx$$

$$W_{slice} = 1000 \cdot \pi \cdot \left(\frac{20-2x}{5}\right)^2 \cdot (9.8) \cdot x \cdot dx$$

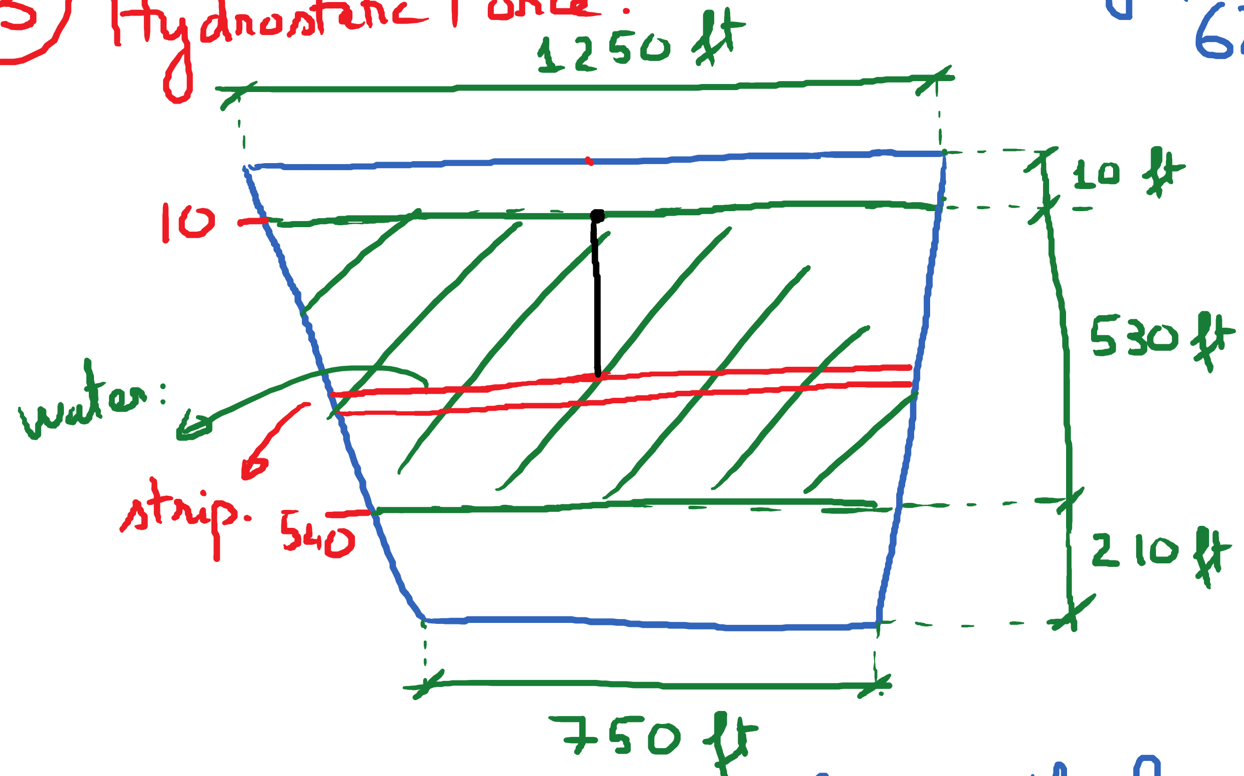
$$= 9800\pi \cdot \left(\frac{20-2x}{5}\right)^2 \cdot x\, dx$$

$$\text{Total work} = \boxed{\int_2^{10} 9800\pi \cdot \left(\frac{20-2x}{5}\right)^2 \cdot x\, dx}$$ ← easy integral.

Thursday, February 1, 2018 2:57 PM

③ Hydrostatic Force.

Density of water: 62.4

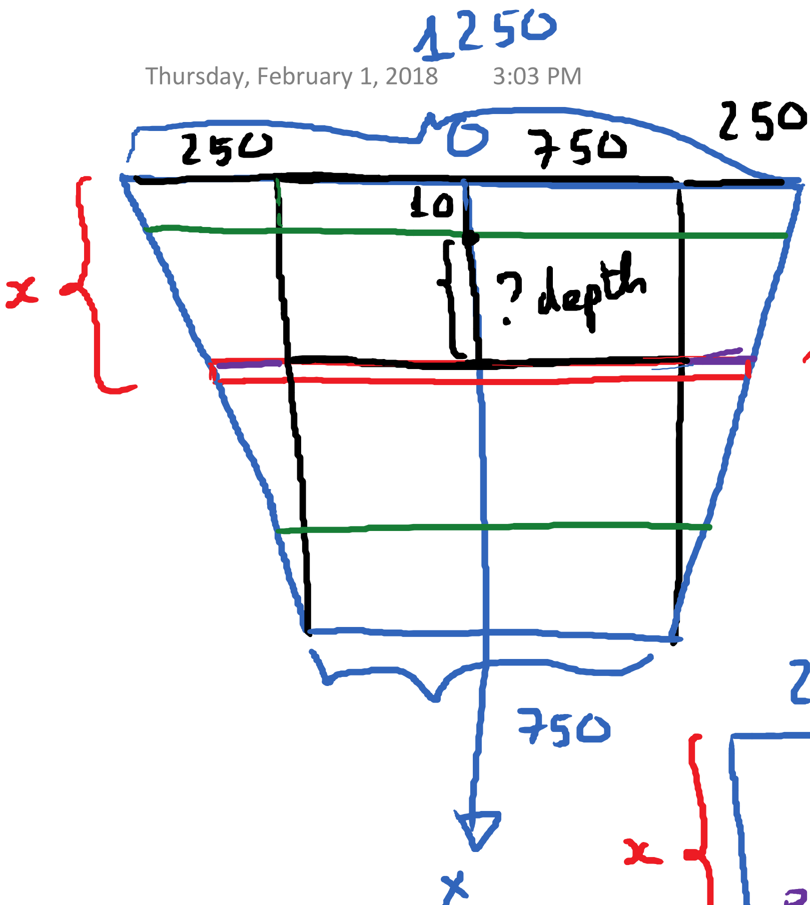


Q: Find the hydrostatic force on the face of the dam.

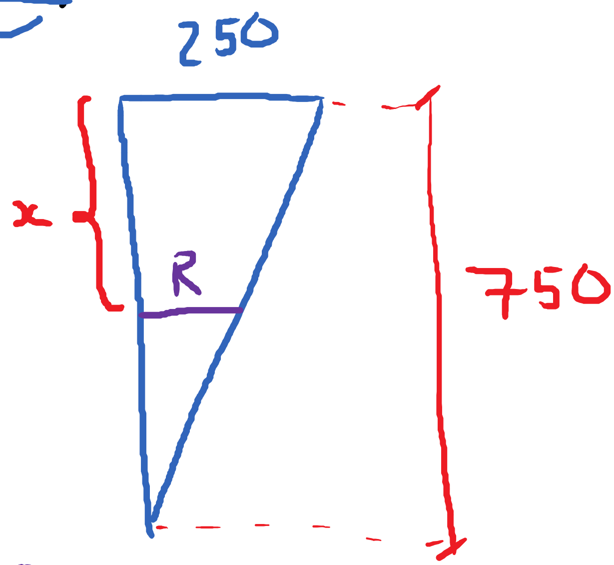
We find the force on the face of a thin strip and integrate

$$\text{Force}_{\text{strip}} = (\text{density}) \cdot (\text{Area of strip}) \cdot (\text{depth})$$

<u>Area of strip.</u>



rectangular strip $= \underbrace{(\text{length})}_{\ell(x)} \cdot \underbrace{(\text{width})}_{dx}$



$$\frac{R}{250} = \frac{750-x}{750}$$

$$R = \frac{750-x}{750} \cdot 250 = \frac{750-x}{3}$$

$$\ell(x) = 750 + 2 \cdot \frac{750-x}{3}$$

$$\ell(x) = 1250 - \frac{2}{3}x$$

$$\text{Area}_{\text{strip}} = \left(1250 - \frac{2}{3}x\right) dx$$

$$\text{Force}_{\text{strip}} = \underbrace{(62.4)}_{\text{density}} \cdot \underbrace{\left(1250 - \frac{2}{3}x\right)}_{\text{area}} \cdot \underbrace{(x-10)}_{\text{depth}} dx$$

density area depth

Total hydrostatic force = $\int_{10}^{540} \text{Force}_{\text{strip}}$