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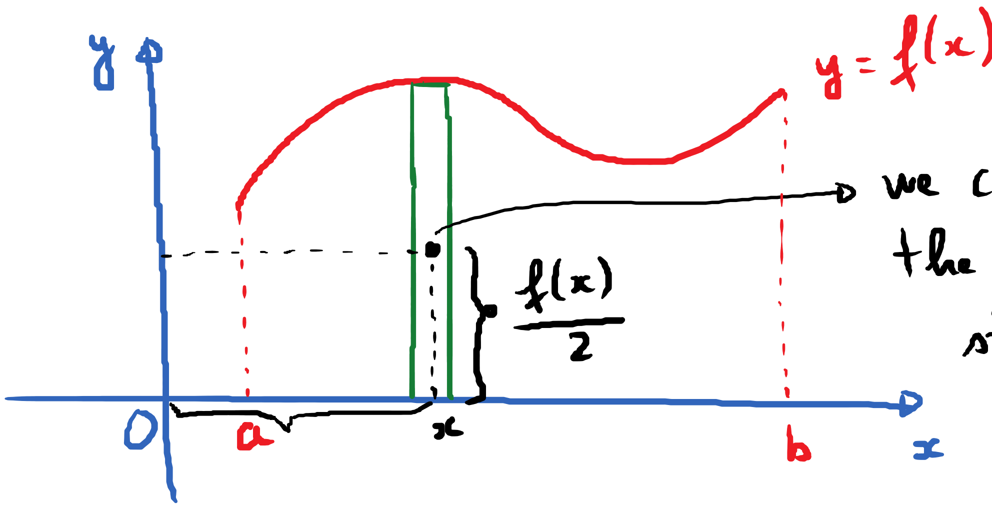
Where does the formula for M_x and M_y come from?

Divide R into "thin" rectangular strips. Back to the case with discrete masses.



we can consider all the masses of this strip to be concentrated at the center.

Center: $\left(x, \frac{1}{2}f(x)\right)$

mass concentrated at $\left(x, \frac{1}{2}f(x)\right) = (\text{density})\cdot(\text{area})$

$= \rho \cdot f(x)\,dx$

y-moment: $\rho\, x\, f(x)\,dx$

x-moment: $\rho \cdot \frac{1}{2} f(x) \cdot f(x)\,dx = \rho \cdot \frac{1}{2}[f(x)]^2 dx$

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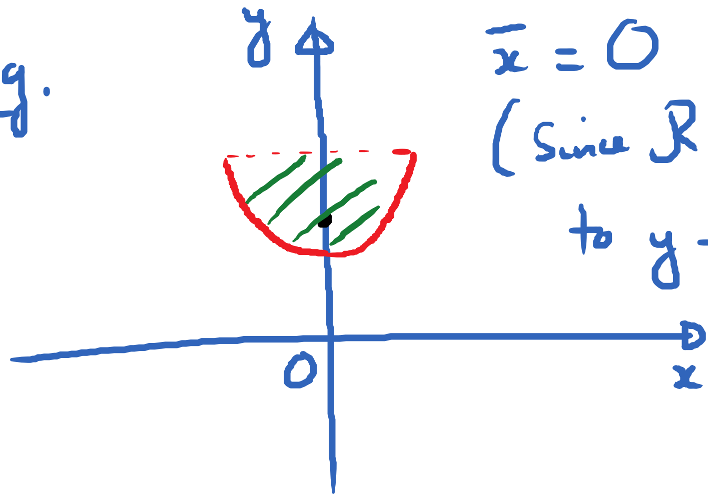
The symmetry Principle.

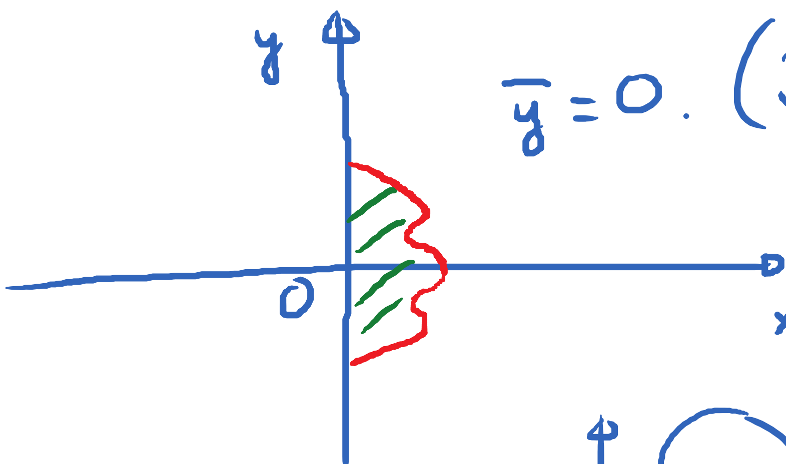
> If a lamina R is symmetric about a line L, then the centroid of R must lie on L.

E.g.

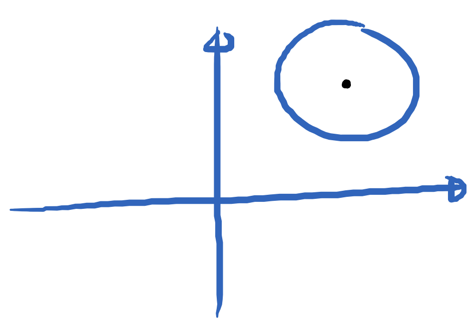


$\bar{x} = 0$

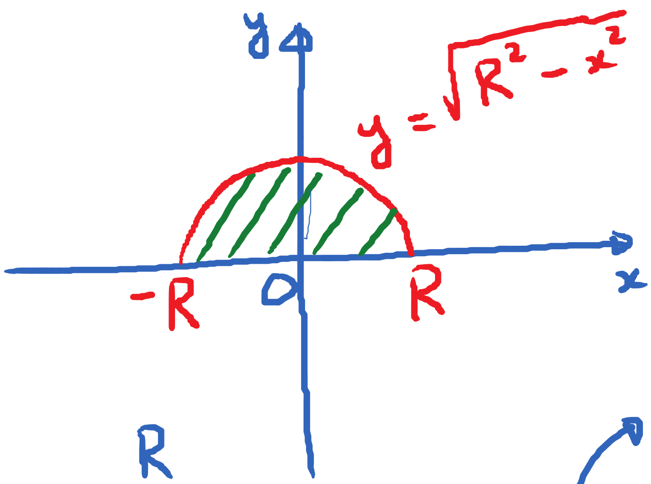
(Since R is symmetric with respect to y-axis)



$\bar{y} = 0$. (R is symmetric with respect to x-axis)



E.g.



Find the centroid $(\bar{x}, \bar{y})$

Since the region is symmetric w.r.t. y-axis, $\boxed{\bar{x} = 0}$

area of semicircle radius R

$$m = \int_{-R}^{R} \sqrt{R^2 - x^2}\, dx = \frac{\pi R^2}{2}.$$

$$M_x = \int_{-R}^{R} \frac{1}{2}\left(\sqrt{R^2 - x^2}\right)^2 dx = \frac{1}{2}\int_{-R}^{R} (R^2 - x^2)\, dx$$

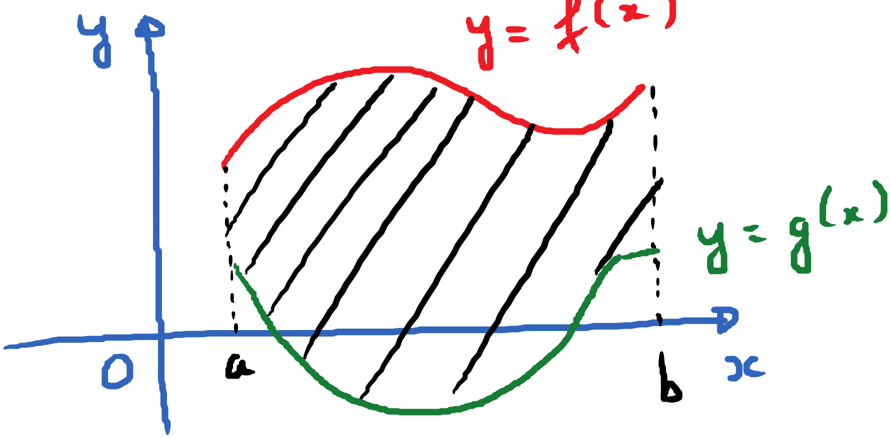
$$= \frac{1}{2}\cdot\left(R^2 x - \frac{x^3}{3}\right)\Bigg|_{-R}^{R}$$

$$= \frac{1}{2}\cdot\left(R^3 - \frac{R^3}{3} + R^3 - \frac{R^3}{3}\right) = \frac{1}{2}\cdot\frac{4R^3}{3} = \frac{2R^3}{3}.$$

$$\bar{y} = \frac{M_x}{m} = \frac{\frac{2R^3}{3}}{\frac{\pi R^2}{2}} = \frac{2R^3}{3}\cdot\frac{2}{\pi R^2} = \frac{4R}{3\pi}.$$

$$\text{Centroid} = \left(0, \frac{4R}{3\pi}\right)$$

* Centroid of lamina bounded by 2 curves.



Find centroid $(\bar{x}, \bar{y})$

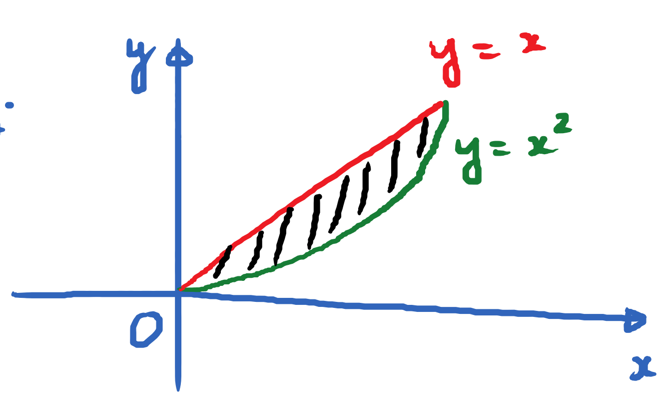
$$\bar{x} = \frac{M_y}{m} \; ; \; \bar{y} = \frac{M_x}{m}$$

$$m = \rho \cdot \int_a^b \left(f(x) - g(x) \right) dx$$

$$M_y = \rho \int_a^b x \cdot \left(f(x) - g(x) \right) dx$$

$$M_x = \rho \int_a^b \frac{1}{2} \left[\left(f(x) \right)^2 - \left(g(x) \right)^2 \right] dx$$

E.g.



Find centroid of this lamina.

Centroid $\left(\frac{1}{2}, \frac{2}{5} \right)$

* Application of Centroid in finding volume of solid of revolution - Theorem of Pappus.

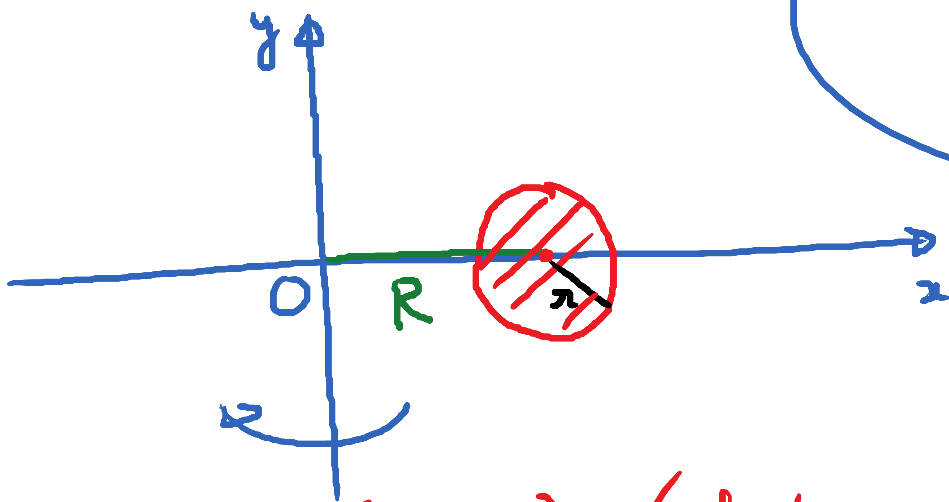
Theorem of Pappus:

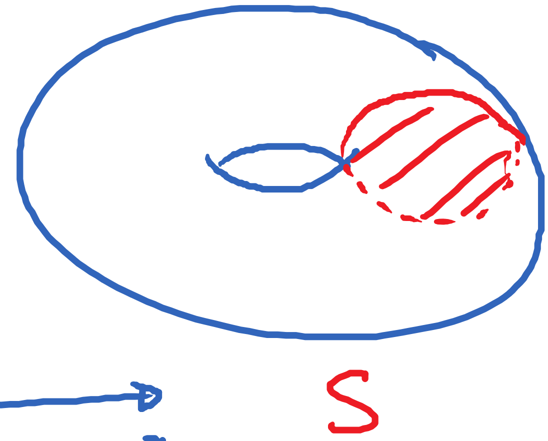
If a solid S is obtained by rotating a region R about an axis, then the volume of S is equal to the product of the area of R and the distance traveled by the centroid of R

$$V_{\text{Solid } S} = (\text{Area of } R) \cdot (d)$$

distance traveled by centroid.

E.g. Volume of doughnut





$$V_S = (\text{Area}) \cdot (\text{distance traveled by centroid})$$

$$= \pi r^2 \cdot 2\pi R$$

$$V_S = 2\pi^2 r^2 R$$