3.1. Integration By Parts

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$$\frac{1}{2}\int 2x \sin(x^2)\, dx \qquad \text{Let } u = x^2. \quad du = 2x\,dx$$

$$\frac{1}{2}\int \sin(u)\, du = -\frac{1}{2}\cos(u) + C$$

$$\boxed{\cancel{\int x\sin(x)\,dx \neq \int x\,dx \cdot \int \sin(x)\,dx}}$$

$\longrightarrow$ Integration by parts.

Integration by parts Formula

$$\boxed{\int u\,dv = uv - \int v\,du}$$

where $u = f(x)$; $v = g(x)$ are continuously differentiable function.

$$\int \underbrace{x}_{u}\ \underbrace{\sin x\,dx}_{dv} \qquad \text{Let} \quad \begin{aligned} u &= x \\ dv &= \sin x\,dx \end{aligned}$$

$$du = 1\,dx = dx \qquad v = \int \sin x\,dx = -\cos x$$

$$\int u\,dv = uv - \int v\,du$$

$$\int x \sin x\,dx = x\cdot(-\cos x) - \int (-\cos x)\,dx$$

$$\int x \sin x\,dx = -x\cos x + \int \cos x\,dx$$

$$\boxed{\int \boxed{x\sin x}\,dx = -x\cos x + \sin x + C}$$

Check the answer:

$$\frac{d}{dx}\left(-x\cos x + \sin x + C\right) = -\left(x\cdot(-\sin x) + \cos x\right) + \cos x$$

$$= x\sin x - \cancel{\cos x} + \cancel{\cos x}$$

$$= x\sin x$$

Where does the integration by parts formula come from?

$$d(uv) = u\,dv + v\,du$$ ← product rule from Cal 1

$$\int d(uv) = \int u\,dv + \int v\,du$$

$$uv = \boxed{\int u\,dv} + \int v\,du$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

E.g. $\int x e^{2x}\,dx \longrightarrow \int u\,dv$

$$\begin{cases} u = x \\ dv = e^{2x}\,dx \end{cases} \longrightarrow \begin{cases} du = dx \\ v = \int e^{2x}\,dx \doteq \frac{1}{2}e^{2x} \end{cases}$$

$$\int \underbrace{x}_{u}\ \underbrace{e^{2x}dx}_{dv} = \underbrace{\frac{1}{2}xe^{2x}}_{uv} - \underbrace{\int \frac{1}{2}e^{2x}dx}_{\int v\,du}$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}dx$$

$$= \boxed{\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C.}$$

Q: How do we know how to choose u and dv?

LIATE rule.

L: Logarithmic Functions. ($\ln x$; $\log_2 x$, ...)

I: Inverse Trig Functions (arcsin x, ... arctan x, ...)

A: Algebraic Functions (polynomials, radicals, ...)

T: Trig Functions (sin x, cos x, ...)

E: Exponential Functions (e^x, 2^x, ...)

Ex. Find the given integrals

(1) $\int x \ln x \, dx$ (2) $\int x \arctan x \, dx$

(3) $\int e^x \cos x \, dx$ (4) $\int x^2 \cos x \, dx$

Short-cut to the integration by parts formula.

$\longrightarrow$ Tabular method for Integration by parts.

(4) $\int x^2 \cos x \, dx$

	D		I	
$u \leftarrow$	x^2	$+$	$\cos x$	$\leftarrow dv$
	$2x$	$-$	$\sin x$	
	2	$+$	$-\cos x$	
	0		$-\sin x$	

Answer:

$$\int x^2 \cos x \, dx = x^2 \sin x - 2x(-\cos x) + 2(-\sin x) + C$$

$$= x^2 \sin x + 2x \cos x - 2\sin x + C$$

3. $\int e^x \cos x \, dx$

	D		I	
$u \rightarrow$	$\cos x$	$+$	e^x	$\leftarrow dv$
	$-\sin x$	$-$	e^x	
	$-\cos x$	$+$	e^x	

$$\int e^x \cos x \, dx = e^x \cos x - e^x(-\sin x) + \int e^x(-\cos x)\, dx$$

$$\int e^x \cos x \, dx = e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$$

$$2 \cdot \int e^x \cos x \, dx = e^x \cos x + e^x \sin x$$

$$\boxed{\int e^x \cos x \, dx = \frac{e^x \cos x + e^x \sin x}{2} + C}$$